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ON THE SURJECTIVITY OF THE WAHL MAP

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1. Introduction. In this paper we will prove a theorem (stated at the end of this introduction) describing the rank of the Wahl map of a general curve of large genus. We begin here by describing this map and some aspects of its significance.

To begin with, consider a smooth curve C, a line bundle L on C, and a linear system $V \subseteq H^0(C, L)$. Given a section $\sigma \in V$ of L, we can try to define a "differential" $d\sigma$ of σ, which will be a section of the tensor product $K \otimes L$ of the canonical bundle $K = K_C$ with L, by choosing a trivializing section σ_0 of C, writing σ locally as

$$\sigma(z) = f(z) \cdot \sigma_0$$

and setting

$$d\sigma = df \otimes \sigma_0.$$

This clearly doesn't work: if τ_0 is another trivializing section on L, with $\sigma_0(z) = g(z) \cdot \tau_0$, we would have

$$\sigma(z) = f(z) \cdot g(z) \cdot \tau_0,$$

so that the "differential" would be

$$d\sigma = (f \cdot dg + g \cdot df) \otimes \tau_0 = df \otimes \sigma_0 + f \cdot dg \otimes \tau_0,$$

i.e., it would differ from the earlier differential $d\sigma$ by the addition of $f \cdot dg \otimes \tau_0$. The expression $d\sigma$ is thus only well defined at the points where σ is zero!

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