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ISOMETRY-INVARIANT GEODESICS ON SPHERES

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Given a Riemannian manifold M with an isometry A, one can look for A-invariant geodesics, i.e., geodesics satisfying $c(t+1) = Ac(t)$. These were first studied by Grove [3]. Probably the simplest example is a rotation of S^2 with the standard metric:

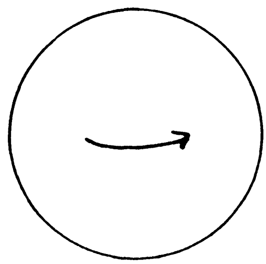

FIGURE 1

Here the only A-invariant geodesic is the equator. This seems to be quite different from the situation where A is the identity, i.e., closed geodesics. There is no known example of a compact Riemannian manifold with only finitely many closed geodesics. It is known that for a generic metric on a sphere there will be infinitely many closed geodesics ([7], [9]). If M is a sphere S^n with the standard metric and $A \in SO(n+1)$, then the A-invariant geodesics on M correspond to 2-planes through the origin in $\mathbb{R}^{n+1}$ which are invariant under A. Such 2-planes always exist; for a generic A there will be only finitely many.

In this paper we show that, on spheres, *the obvious examples are pathological* in the following sense:

> *Let M be a sphere with the standard metric g and A a rotation of finite order. Then g lies in the closure of the interior of the set S of A-invariant metrics with infinitely many A-invariant geodesics.*

Grove and Tanaka proved the following theorem: Let M be a compact, simply connected Riemannian manifold and let A be an isometry of M. If the Betti numbers of the space of A-invariant curves on M are unbounded, then M has infinitely many invariant geodesics. (For A the identity this is the theorem of Gromoll-Meyer; the theorem was proved for A of finite order in [6] and for A

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