Vol. 57, No. 3 DUKE MATHEMATICAL JOURNAL © December 1988

THE FUNDAMENTAL GROUP OF A RIEMANN SURFACE: MIXED HODGE STRUCTURES AND ALGEBRAIC CYCLES

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1. Introduction. Suppose X is a compact Riemann surface of genus $g > 1$. For each $p \in X$, the integral group ring of the fundamental group $\mathbf{Z}\pi_1(X, p)$ is a natural topological invariant of the pointed space (X, p). Denote by J_p the kernel of the ring homomorphism

$$\mathbf{Z}\pi_1(X, p) \to \mathbf{Z}$$

$$\Sigma n_1\gamma_1 \mapsto \Sigma n_i.$$

According to J. Morgan [17], there is a mixed Hodge structure on the truncated group ring $\mathbf{Z}\pi_1(X, p)/J_p^3$. This mixed Hodge structure is an analytic invariant of the pointed curve (X, p). It is natural to ask to what extent it determines the curve and its base point.

To examine this question, we use ideas of J. Carlson, R. Hain, and J. Morgan on extensions of mixed Hodge structures and mixed Hodge structures on fundamental groups. The mixed Hodge structure on (X, p) is calculated using iterated integrals and is strongly dependent on the base point. The formula obtained to describe this mixed Hodge structure is very similar to a formula introduced by B. Harris in his paper "Harmonic Volumes" [13], where he used iterated integrals to describe the Abel-Jacobi image of the 1-cycle $X - X^-$ in Jac(X). The pointed harmonic volume I_p described in the present paper was first introduced in Harris's paper, though the emphasis there was on a restriction of I_p which was independent of base point. We will employ many of Harris's constructions, taking more careful note of the base point.

The main result of this paper is the following:

THEOREM. *Suppose that X and Y are smooth projective curves and that $p \in X$ and $q \in Y$. With the possible exception of two points p in X, if there is a ring isomorphism*

$$\mathbf{Z}\pi_1(X, p)/J_p^3 \to \mathbf{Z}\pi_1(Y, q)/J_q^3$$

which preserves the mixed Hodge structure, then there is a biholomorphism φ: $X \to Y$ such that $\varphi(p) = q$.

Received October 28, 1986. Revision received February 24, 1988.