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SELF-INTERSECTION 0-CYCLES AND COHERENT SHEAVES ON ARITHMETIC SCHEMES

TAKESHI SAITO

Introduction. In arithmetic geometry, the conductor of the ℓ-adic étale cohomology is one of the most important numerical invariants of an arithmetic scheme. In his recent paper [2] Bloch defined the self-intersection 0-cycle $(\Delta_X, \Delta_X)_S$ of an arithmetic scheme X relative to a base scheme S, to study the conductor. There he conjectured that its degree is equal to the conductor of X and proved it in relative dimension 1. On the other hand, Kato conjectured that the conductor is equal to the alternating sum of the Euler-Poincaré characteristics of the torsion parts of the sheaves of relative differentials $\Omega^{\bullet}_{X/S}$. The main purpose of this paper is to show that these two conjectures are equivalent. Namely, we prove the equality between the degree of $(\Delta_X, \Delta_X)_S$ and the alternating sum of the Euler-Poincaré characteristic of the torsion part of $\Omega^{\bullet}_{X/S}$.

Let S be the spectrum of a discrete valuation ring with perfect residue field and X be a regular flat proper S-scheme purely of relative dimension r with smooth generic fiber. Then the conductor $\mathrm{Art}(X/S)$ of X over S is defined to be the integer $\chi(X_{\bar\eta}) - \chi(X_{\bar s}) + \mathrm{Sw}_S H^*(X_{\bar\eta}, \mathbf{Q}_\ell)$ (cf. [2], section 0). Here ℓ is a prime invertible on S, s (resp. η) denotes the closed (resp. generic) point of S, and Sw denotes the Swan conductor.

By the definition of Bloch ([2], section 1), the self-intersection 0-cycle $(\Delta_X, \Delta_X)_S$ is the localized chern class $(-1)^{r+1}c^X_{r+1, X_s}(\Omega^1_{X/S}) \in CH_0(X_s)$. Then he made the following

CONJECTURE (Bloch). *For X over S as above,*

$$-\mathrm{Art}(X/S) = \deg(\Delta_X, \Delta_X)_S.$$

On the other hand, the supports of the coherent sheaves $\Omega^p_{X/S,\mathrm{tors}}$ are contained in the closed fiber X_s. For a coherent $\mathcal{O}_X$-module $\mathcal{F}$ whose support is in X_s, the Euler-Poincaré characteristic $\chi(X, \mathcal{F})$ denotes the alternating sum of the lengths of the $\mathcal{O}_S$-modules of finite length $H^q(X, \mathcal{F})$. Then Kato made the

CONJECTURE (Kato). *For X over S as above,*

$$-\mathrm{Art}(X/S) = \sum_{p=0}^{r+1} (-1)^p \chi\left(X, \Omega^{\bullet}_{X/S,\mathrm{tors}}\right).$$

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