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A CHARACTERIZATION OF BALL QUOTIENTS WITH SMOOTH BOUNDARY

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1. Introduction. In 1977 S.-T. Yau proved that a compact Kähler manifold with the negative or zero first Chern class admits a Kähler-Einstein metric [16] (T. Aubin contributed also to the result in the case of the negative first Chern class ([1]).) As an application of this existence theorem, he proved that a compact Kähler manifold X of dimension n with the negative first Chern class satisfies the inequality

$$(-1)^n c_1^n(X) \leqslant (-1)^n \frac{2(n+1)}{n} c_1^{n-2}(X) c_2(X)$$

and the equality holds if and only if X is a compact unramified quotient of the unit ball in $\mathbf{C}^n$.

The purpose of this article is to give a characterization of toroidal compactifications of unramified quasi-projective ball quotients with smooth boundary. This is a continuation of my work [14].

THEOREM 1. *Let X be a projective algebraic manifold of dimension n defined over $\mathbf{C}$ and let D be a smooth divisor on X. Assume that*

1. *$K_X + (1-\varepsilon)D$ is ample for every sufficiently small positive rational number ε;*
2. *$K_X + D$ is numerically trivial on D;*
3. *$K_X + D$ is ample modulo D and semiample (cf. Definitions 1, 2).*

Then the inequality

$$c_1^n\big(\Omega_X^1(\log D)\big) \leqslant \frac{2(n+1)}{n} c_1^{n-2}\big(\Omega_X^1(\log D)\big) c_2\big(\Omega_X^1(\log D)\big)$$

holds and the equality holds if and only if $X - D$ is an unramified quotient of the unit ball in $\mathbf{C}^n$.

Remark 1. A toroidal compactification of an unramified arithmetic quotient of the unit ball in $\mathbf{C}^n$ with smooth boundary satisfies the condition of Theorem 1. This follows from the fact that the canonical Kähler-Einstein form represents the current which is cohomologous to 2π times the logarithmic canonical class on the toroidal compactification of the ball quotient with small boundary and the form

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