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HOMOGENEOUS SPACES WITHOUT TAUT EMBEDDINGS

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A large number of homogeneous spaces have been shown to admit embeddings as taut submanifolds. All such examples known to the author are the standard embeddings of the so-called R-spaces, which were shown to be taut in [16], or, equivalently, the orbits of isotropy representations of symmetric spaces, which are taut by [2]. Many proofs have been given of tautness of special classes of R-spaces where the arguments are easier. These include the spheres, the projective spaces, more generally the Grassmannians, and also the unitary groups; see [8]. Further examples are compact homogeneous Kähler manifolds [7], extrinsically symmetric spaces [5], and homogeneous isoparametric hypersurfaces and their focal submanifolds [3]. Finally, there are the homogeneous isoparametric submanifolds with higher codimension and their focal submanifolds [6], which are exactly the R-spaces.

Examples of homogeneous spaces not admitting any taut embeddings have apparently not appeared in the literature. It is the purpose of the present paper to find conditions on compact homogeneous spaces which guarantee that there is an abundance of such examples. This will be a consequence of the following theorem:

THEOREM. *Let $M \subset E^N$ be a taut submanifold with respect to a field F and let $i > 0$ be the smallest number such that $H_i(M; F)$ is nontrivial. Then every torsion element in $H_i(M; \mathbb{Z})$ is of order two. In particular, $H_i(M; \mathbb{Z})$ is without torsion if the characteristic of F is not two.*

We first apply the theorem to the lens spaces $L(p, q)$ which are homogeneous for $q = 1$. Since $H_1(L(p, q); \mathbb{Z}) = \mathbb{Z}_p$, one sees that the real projective space $L(2, 1)$ is the only lens space that can admit a taut embedding by the theorem. It is well known that the projective space admits taut embeddings. Thus we have the following corollary:

COROLLARY. *The only lens space admitting taut embeddings is the real projective space.*

One also sees that a taut compact 3-manifold is always taut with respect to $\mathbb{Z}_2$. The proof of the theorem shows that this is also true in the noncompact case.

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