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ON L^2 WELL POSEDNESS OF THE CAUCHY PROBLEM FOR SCHRÖDINGER TYPE EQUATIONS ON THE RIEMANNIAN MANIFOLD AND THE MASLOV THEORY

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0. Introduction. Let M be a C^∞ complete Riemannian manifold without boundary. M may be nonorientable. In the present paper we consider the Cauchy problem for Schrödinger type equations on M

$$(0.1)\quad \begin{cases} Lu(t,Q) \equiv \left[\dfrac{1}{i}\partial_t - \dfrac{1}{2}\Delta_M + \mathbf{B} + C(Q)\right]u(t,Q) = f(t,Q) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (Q \in M), \\ u(0,Q) = u^{(0)}(Q), \end{cases}$$

where Δ_M denotes the Laplace-Beltrami operator on M, $C(Q)$ a C^∞ function on M and $\mathbf{B}$ belongs to the complexification of the space of all C^∞ vector fields on M. $\mathbf{B}u(t,Q)$ denotes the Lie derivative. We assume that

$$(0.2)\qquad M \text{ has a countable base.}$$

There, a family of open sets $\{\mathcal{O}_i\}_{i=1}^\infty$ on M is called a countable base, if for an arbitrary open set $\mathcal{O}$ on M and an arbitrary point $p \in \mathcal{O}$ there is an index j such that $p \in \mathcal{O}_j \subset \mathcal{O}$.

We know from [12] that the Cauchy problem (0.1) has an infinite propagation speed. So, it is impossible to consider the well posedness of (0.1) in $C^\infty(M)$ space. Therefore, in the present paper we shall consider the well posedness of (0.1) in the sense of L^2. Let dV_M be the volume element associated with the Riemannian metric $g\langle \cdot , \cdot \rangle$ on M in a classical sense and we denote by $L^2 \equiv L^2(M)$ the set of all square integrable functions on M

$$\left\{ f(Q);\ \int_M |f(Q)|^2\, dV_M < \infty \right\}.$$

The precise definitions of these will be given in section 1. We also denote the set of all L^2 valued continuous function in $t \in [0,T]$ by $\mathscr{E}_t^0([0,T]; L^2)$. The solution $u(t,Q)$ of (0.1) is considered in a distribution sense (Definition 1.3). Then, we adopt the following definition as in [7].

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