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MAPPINGS OF THREE-DIMENSIONAL CR MANIFOLDS AND THEIR HOLOMORPHIC EXTENSION

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I. Introduction

§1. Introduction and main results. A smooth manifold M is called a *CR (Cauchy–Riemann) manifold* if there is a subbundle $\mathcal{V}$ (called the *CR bundle*) of $\mathbb{C}TM$, the complexified tangent bundle of M, satisfying $[\mathcal{V}, \mathcal{V}] \subset \mathcal{V}$ and $\mathcal{V} \cap \overline{\mathcal{V}} = \{0\}$. If M and M' are CR manifolds with CR bundles $\mathcal{V}$ and $\mathcal{V}'$, a smooth mapping $H\colon M \to M'$ is called CR if for every $p \in M$,

$$H'(\theta) \in \mathcal{V}'_{H(p)} \tag{1.1}$$

for all $\theta \in \mathcal{V}_p$, the fiber of $\mathcal{V}$ at p. Here $H'\colon \mathbb{C}TM \to \mathbb{C}TM'$ is the differential map of H. If M and M' are three-dimensional, then necessarily, $\dim_{\mathbb{C}} \mathcal{V} = \dim_{\mathbb{C}} \mathcal{V}' = 1$.

Locally, near $p_0 \in M$ and $p_0' = H(p_0) \in M'$, there exist smooth nonvanishing vector fields L and L', sections of $\mathcal{V}$ and $\mathcal{V}'$, respectively. Condition (1.1) can then be written

$$H'(L_p) = \lambda(p) L'_{H(p)} \tag{1.2}$$

for some smooth function λ defined on M near p_0.

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