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CARLEMAN'S AND SUBELLIPTIC ESTIMATES

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1. Introduction. The main goal of this paper is to give a positive answer on a conjecture of Treves [9] that has been partially investigated by Menikoff [7]. Let us first recall what is meant by a Carleman estimate, a very useful tool in proving uniqueness properties for semilinear PDE. Let s be a real number and γ a "parameter" larger than 1. We set (e.g., for $u \in \mathscr{S}(\mathbb{R}^n)$)

$$\|u\|_{s,\gamma} = \left(\int (\gamma^2 + |\xi|^2)^s |\hat{u}(\xi)|^2 \, d\xi\right)^{1/2}. \tag{1.1}$$

Note that $\|u\|_{0,\gamma} = \|u\|_{L^2}$, and that if s is a positive integer, $\|u\|_{s,\gamma}$ is equivalent to $\gamma^s \|u\|_{L^2} + \|u\|_{H^s}$ or

$$\sum_{j=0}^{s} \gamma^{s-j} \|u\|_{H^j}$$

(uniformly with respect to u and $\gamma \geqslant 1$). Let P be a differential operator of order m in Ω open set of $\mathbb{R}^n$, and ψ a smooth real-valued function. We set

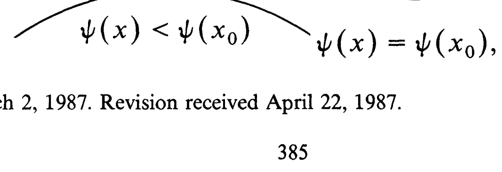
$$P_\gamma = e^{-\gamma\psi} P e^{\gamma\psi} \qquad (\gamma \geqslant 1). \tag{1.2}$$

A Carleman estimate with loss $\delta = k/k+1$ will be

$$\gamma^{1/k+1} \|u\|_{m-1,\gamma} \leqslant C\left(\|P_\gamma u\|_{L^2} + \|u\|_{m-1,\gamma}\right), \tag{1.3}$$

satisfied for $\gamma \geqslant \gamma_0 \geqslant 1$ and $u \in C_0^\infty(K_0)$, K_0 compact $C\Omega$.

Such an estimate is useful to prove the (local forward) uniqueness for the Cauchy problem across any (oriented) hypersurface S such that the level surface of ψ is as follows:



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