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THE LOCAL BEHAVIOUR OF WEIGHTED ORBITAL INTEGRALS

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Introduction. Let G be a reductive algebraic group over a local field F of characteristic 0. The invariant orbital integrals

$$J_G(\gamma, f) = |D(\gamma)|^{1/2} \int_{G_\gamma(F)\backslash G(F)} f(x^{-1}\gamma x)\, dx, \qquad \gamma \in G(F),\ f \in C_c^\infty(G(F)),$$

are obtained by integrating f with respect to the invariant measure on the conjugacy class of γ. They are of considerable importance for the harmonic analysis of $G(F)$. Invariant orbital integrals are also of interest because they occur on the geometric side of the trace formula, in the case of compact quotient. For the general trace formula, the analogous terms are weighted orbital integrals [3]. They are obtained by integrating f over the conjugacy class of γ, but with respect to a measure which is not in general invariant. Weighted orbital integrals may also play a role in the harmonic analysis of $G(F)$, but this is not presently understood. Our purpose here is to study the weighted orbital integrals as functions of γ. In particular, we shall show that they retain some of the basic properties of ordinary orbital integrals.

Recall a few of the main features of the invariant orbital integrals. If F is an Archimedean field, they satisfy the differential equations

$$(1) \qquad J_G(\gamma, zf) = \partial(h_T(z)) J_G(\gamma, f), \qquad \gamma \in T_{\mathrm{reg}}(F),$$

where $T_{\mathrm{reg}}(F)$ is the set of regular points in a maximal torus of $G(F)$, z is an element in the center of the universal enveloping algebra, and $\partial(h_T(z))$ is the corresponding invariant differential operator on $T(F)$. If F is a p-adic field, there are no differential equations. Instead, one has the Shalika germ expansion

$$(2) \qquad J_G(\gamma, f) = \sum_{u \in (\mathcal{U}_G(F))} \Gamma(\gamma, u) J_G(u, f)$$

about 1, or more generally about any semisimple point in $G(F)$. The coefficients $\{\Gamma(\gamma, u)\}$ are functions of regular points γ near 1 and are indexed by the unipotent conjugacy classes u in $G(F)$. If F is either Archimedean or p-adic, the

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