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ERRATUM: "COMMENSURABILITY OF CO-COMPACT THREE-DIMENSIONAL HYPERBOLIC MANIFOLDS"

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It has been pointed out to me by Colin Maclachlan and Alan Reid that Theorem 4.2 of my paper with the above title is incorrect. Their counterexample is as follows:

Let $\Gamma_0(3) = \left\{\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2, \mathbb{Z}(i)),\ \beta \equiv 0 \pmod 3\right\}$. Let $\Gamma_1(3) = \langle \Gamma_0(3), A \rangle$, where $A = \begin{pmatrix} \sqrt{3} & 2\sqrt{3} \\ \dfrac{1}{\sqrt{3}} & \sqrt{3} \end{pmatrix}$. Then $\Gamma_1(3)$: $\Gamma_0(3) = 2$, but their trace fields are distinct.

Since this group is not co-compact, it is not a counterexample to the theorem as stated; but it does have finite co-volume, so the arguments in my proof, if correct, would have applied equally.

The argument is faulty because I tried to apply Mostow's theorem to $SL(2, C)$ instead of to $PSL(2, C)$. All that we can say about an isomorphism σ of k fixing k_1 would be that $A^\sigma = \pm A$. Thus, k might be a Pythagorean extension of k_1, i.e., obtained by the adjunction of one or more square roots. Since we can easily construct infinitely many cyclotomic fields no two of which are Pythagorean extensions of a common subfield, a modification of my argument will still provide an infinity of commensurability classes. Details will be given in a later paper.

Colin Maclachlan and Alan Reid deserve thanks for drawing my attention to this rather subtle error. Colin Maclachlan has also indicated to me that he has an alternative proof, using quaternion algebras, of the existence of an infinity of commensurability classes.

REFERENCES

1. A. M. MACBEATH, *Commensurability of co-compact three-dimensional hyperbolic groups*, Duke Math. J. **50** (1983), 1245–1253.