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THE Γ-FUNCTION IN THE ARITHMETIC OF FUNCTION FIELDS

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Introduction. Let $\Gamma(s)$ be Euler's gamma-function. Over the years this function, which arose out of a desire to have a continuous "factorial," has come to play a central role in the theory of numbers. It is of interest not only because for the Riemann zeta-function, $\zeta(s)$, we have

$$\Gamma(s)\zeta(s) = \int_0^\infty t^{s-1}(e^t - 1)^{-1}\,dt,$$

but also for the properties that it possesses alone. Among these properties are a multiplication formula which describes $\Gamma(ns)$, $n \in \mathbb{N}$, and a functional equation which connects $\Gamma(s)$ with the trigonometric functions and the period 2π. Moreover, it has recently been found that the special values of $\Gamma(s)$ possess remarkable properties such as Deligne's algebraicity formula, which establishes that a certain product of special values generates an Abelian extension of $\mathbb{Q}(\zeta_p)$. A p-adic version of $\Gamma(s)$ has also been constructed by Morita and found to be related to Gauss sums (i.e., the result of Gross–Koblitz). For a good introduction into the early history of the gamma function the reader can consult ([D1]).

In the arithmetic theory of function fields over a finite field, L. Carlitz constructed a "factorial polynomial" Γ_i in order to be able to prove a version of the well-known theorem of von Staudt on Bernoulli numbers (see, for example, [G1]). This factorial has all the divisibility properties one would expect from the analogy to $n!$. By manipulating these values so as to make them *units* (à la Morita) at a prime w of the rational function field $\mathbf{k}$, these values can also be interpolated p-adically to a continuous gamma function, $\Gamma_w(z)$, with values in the completion $\mathbf{k}_w$, of $\mathbf{k}$ at w (see [G3]).

This paper is devoted to an exposition of recent developments in the theory of these Γ-functions. The first of all these, due primarily to the fundamental work of Dinesh Thakur, but with a very important contribution by E.-U. Gekeler, establishes that $\Gamma_w(z)$ possesses *all* of the properties associated with the classical Γ-functions *outside* of the above relationship with $\zeta(s)$. Thus Thakur establishes that $\Gamma_w(z)$ has a multiplication formula (for n prime to the characteristic of the

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