Vol. 56, No. 1 DUKE MATHEMATICAL JOURNAL © February 1988

ON HOLOMORPHIC FUNCTIONS IN THE BALL WITH ABSOLUTELY CONTINUOUS BOUNDARY VALUES

PATRICK AHERN AND JOAQUIM BRUNA

1. Introduction. We start by recalling some well-known facts about the boundary regularity of holomorphic functions in the open unit disc Δ satisfying a growth condition of type $f' \in H^p$, i.e.,

$$\|f'\|_p^p = \sup_{0<r<1} \int_0^{2\pi} |f'(re^{i\theta})|\, d\theta < +\infty.$$

For $p = 1$, it is usually attributed to Privalov that any such function is continuous up to the boundary and, moreover, that the boundary function is absolutely continuous ([6, Thm. 3.12]). In fact, the result is essentially equivalent to the F. and M. Riesz theorem, and other texts attribute it to Hardy and Littlewood. More can be said about f, for Hardy's inequality ([6, p. 487]) implies that if

$$f(z) = \sum_{k=0}^{\infty} a_k z^k,$$

then

$$\sum_{k=0}^{\infty} |a_k| \leqslant \pi \|f'\|_1,$$

that is, the Taylor expansion is norm-convergent in L^∞.

One can view the situation above as the limiting case of two other results, both due to Hardy and Littlewood: if $f' \in H^p$ with $p < 1$, then $f \in H^q$ with $q = p/(1-p)$, and if $f' \in H^p$ with $p > 1$, then f satisfies a Hölder condition with exponent $1 - 1/p$ ([6, Thm. 5.12 and Ex. 9, p. 91]).

To describe the several-variables situation, let us introduce first some notation. We will denote by B^n the unit ball of $\mathbb{C}^n$, by S its boundary, and by $H^p(B^n)$ the

Received May 27, 1986. Revision received April 9, 1987. First author partially supported by an NSF grant. Second author partially supported by grant no. 1593-82 of the Comissón Asesora de Investigación Científia y Técnica, Madrid.