COHOMOLOGY OF FIBER SYSTEMS AND MORDELL–WEIL GROUPS OF ABELIAN VARIETIES

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Introduction. Fiber systems of abelian varieties, constructed from symplectic representations of algebraic groups, have been extensively studied (see, for example, the papers of Shimura, Kuga, Mumford, and Satake in section 4 of [1]). If ρ: $G \to \mathrm{Sp}(V, E)$ is a faithful representation of a real semisimple Lie group G, inducing a holomorphic map $D \to D'$ on associated hermitian symmetric domains (see section 1 for details) and Γ is an arithmetic (sufficiently small) subgroup of G, leaving invariant a lattice L in the real vector space V (viewed as a Γ-module via ρ), one can obtain a (holomorphic) fiber system $W \cong (D \times V)/(\Gamma \ltimes L) \to \Delta \cong D/\Gamma$ with abelian varieties as fibers, and with Δ and W realizable as complex quasiprojective varieties. The fiber A over the generic point of Δ is an abelian variety defined over $\mathbb{C}(\Delta)$, the function field of Δ. For $G = \mathrm{SL}_2(\mathbb{R})$, Γ a subgroup of finite index in $\mathrm{SL}_2(\mathbb{Z})$, $-1 \notin \Gamma$, and ρ the standard representation on $V = \mathbb{R}^2$, Shioda ([16]) showed $A(\mathbb{C}(\Delta))$ is finite. The finiteness of $A(\mathbb{C}(\Delta))$ has also been shown when the fiber system is characterized by certain endomorphism algebra structures, in [17] and [18]. In this paper (see Proposition 4 and Theorem 5) we give general criteria for the finiteness of $A(\mathbb{C}(\Delta))$:

$$\textit{If } H^1(\Gamma, V) = 0 = H^0(\Gamma, V), \textit{ then } A(\mathbb{C}(\Delta)) \textit{ is finite.}$$

Using these criteria, one obtains the finiteness of the Mordell–Weil group $A(\mathbb{C}(\Delta))$ in great generality. One example in which the criteria hold is when $n > 1$, $V = \mathbb{R}^{2n}$, $L = \mathbb{Z}^{2n}$, ρ is the standard representation of $G = \mathrm{Sp}(n, \mathbb{R})$ on V, and $\Gamma \subset \mathrm{Sp}(n, \mathbb{Z})$ is the principal congruence subgroup of level $N \geq 3$. In this case, Shioda conjectured in 5.7 of [16] that $A(\mathbb{C}(\Delta)) \cong (\mathbb{Z}/N\mathbb{Z})^{2n}$. This was proved in [17] by different methods. The finiteness of $A(\mathbb{C}(\Delta))$ was also shown in [17] and [18] in several important cases in which the criteria are not satisfied, including cases where (1) $G \cong SL_2(\mathbb{R})$ and Γ is cocompact, and (2) $G \cong SU(n, 1)$.

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1. Fiber systems of abelian varieties. (See [13].) Suppose V is a vector space over $\mathbb{R}$ of dimension $2n$, E is a nondegenerate alternating bilinear form on V,

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