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SPREADING OF SINGULARITIES AT THE BOUNDARY IN SEMILINEAR HYPERBOLIC MIXED PROBLEMS I: MICROLOCAL $H^{s,s'}$ REGULARITY

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1. Introduction and statement of results. Solutions $u \in H^s_{\text{loc}}(R^n)$, $s > n/2$, of nonlinear strictly hyperbolic equations with singularities in the past will in general develop "anomalous" singularities, that is, singularities not present in the solutions to corresponding linear problems. Much work has been done since the late seventies on the question of describing the location and strength of anomalous singularities for problems posed in free space. Bony [7] and Rauch [15] showed that the new singularities will always be at least twice as weak (roughly) as the original singularities, no stronger than about $H^{2s-n/2}$. Two mechanisms for the production of anomalous singularities have been identified. They are created when singularity-bearing characteristics cross; new singularities can then emerge along all other forward characteristics issuing from the crossing point (Rauch–Reed [16], [17]; Beals [2]). In addition, a single singularity-bearing characteristic can give rise to anomalous singularities through self-spreading (Beals [3]). For equations of order higher than 2, singularities of strength $\sim 2s - n/2$ do actually, in general, appear [16]. Hence, it is striking that for second-order strictly hyperbolic equations in free space, anomalous singularities of strength $\sim 2s - n/2$ never appear. When $n > 2$, Beals ([3], [4]) has shown that for such equations, microlocal regularity up to order roughly $3s - n$ propagates along null bicharacteristics. So anomalous singularities in these cases have strength at most $3s - n$. (For $n = 2$ see Reed [17].)

Until recently few results were available, except for the case $n = 2$ (e.g., [6], [14]), about nonlinear propagation on domains $\Omega \subset R^n$ with boundary. The papers of Alabidi [1] and Sablé-Tougeron [18] showed, for solutions to a large class of fully nonlinear boundary problems, that microlocal regularity up to order $\sim 2s - n/2$, propagating along a bicharacteristic that meets $\partial\Omega$ transversally, continues along the reflected bicharacteristic. In David–Williams [9] (see also [8]) a $2s$ theorem of this type was proved for second-order, semilinear problems with Dirichlet conditions in the general case where, in addition to transversal rays, bicharacteristics tangent to $\partial\Omega$ to arbitrarily high finite or infinite order as well as gliding rays may carry singularities. Leichtnam [11] has obtained a $2s$ result describing propagation along generalized bicharacteristics for fully nonlinear second-order equations with Dirichlet conditions. These theorems imply that anomalous singularities in boundary problems have strength at most $\sim 2s - n/2$, just as in free space.

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