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BOUNDING LAMINATIONS

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§0. Introduction. In [5], an algorithm was introduced to decide whether a given surface automorphism was nullcobordant, that is, whether it "extended" over some compact 3-manifold. The method was to show that it is possible to decide whether an automorphism "compresses." The question was reduced to the consideration of the case that the map was pseudo-Anosov and exploited certain convergence properties of pseudo-Anosov maps and their invariant laminations. In this paper we abstract the notion of convergence used there to more general laminations in the hope that this will provide some insight to the nature of these problems. A by-product of our methods is some new results concerning the cobordism group of surface automorphisms.

Our notion of convergence for laminations comes from the following definition: A pair of minimal filling laminations bounds in a compression body if they are the limit in the Hausdorff topology of the boundaries of discs embedded in the compression body. In the particular case that the lamination is a simple closed curve, this definition of convergence forces the curve to bound a disc in the compression body.

Our approach is based on an interplay between regular planar coverings of a surface and compression bodies and leads to the notion of a "strongly homoclinic" leaf, which acts as the analogue of a closed curve in the case that the lamination has no closed leaves. This characterization leads to our first main result: a finiteness result for laminations which bound in this sense:

THEOREM. *A pair of minimal transverse laminations* (L, μ) *and* $(L', \mu') \subset F$ *bounds in only finitely many minimal compression bodies.*

Again, this result extends the analogous result for pairs of closed curves bounding discs in a compression body.

We conclude with applications to the cobordism group of surface automorphisms. For example, it follows from this finiteness result that if a pseudo-Anosov has invariant laminations that bound in this sense, then some power of it extends. Further, in the case of genus 2, the situation in which a map does not extend but some power of it does can only happen finitely often:

THEOREM. *Let* $\theta : F \to F$ *be a pseudo-Anosov map of a genus-2 surface, which does not extend over any compression body. Then* θ^p *does not extend for all sufficiently large primes* p.

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