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SUPPORT THEOREMS FOR REAL-ANALYTIC RADON TRANSFORMS

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1. Introduction. A generalized Radon transform, R, integrates functions on a manifold X over each member of a class of submanifolds, Y, using a specified measure of integration for each submanifold in Y. For both practical [Tretiak] and theoretical reasons [Helgason 1973], fairly arbitrary classes of submanifolds and fairly arbitrary measures are considered. Gelfand [1966], Helgason [1970], Grinberg [1983], Quinto [1983], and many others have investigated these transforms using techniques from group representations to integral equations. Our work is based on the seminal work of Guillemin. Guillemin and Sternberg [1977] have proven that many generalized Radon transforms are elliptic Fourier integral operators and that composition with their adjoints, R^*R, are elliptic pseudodifferential operators in the C^∞ category. The role the measures play in this pseudodifferential operator has been investigated [Quinto 1980]. Guillemin and Sternberg [1979] have proven range theorems for these transforms, and Guillemin [1985] has proven the characterization of admissible line complexes in C^3 using microlocal analysis. If the submanifolds and the measures are real-analytic, R^*R has been proven to be an analytic pseudodifferential operator in certain cases, which implies invertibility [Boman 1984, 1986]. In contrast, Boman [1985] discovered counterexamples to invertibility for positive C^∞ measures.

Many Radon transforms satisfy support theorems. Given appropriate functions f and appropriate subsets A of Y, if $Rf(y) = 0$ for each submanifold y in A, then f is zero on the union of the submanifolds in A. Helgason [1965, 1973] proved this for many group-invariant Radon transforms, including the classical transform integrating over hyperplanes in $\mathbb{R}^n$ in Lebesgue measure. Cormack [1981, 1982], Solmon [1976], and others have proven support theorems for transforms integrating over various curves and surfaces and with nonstandard measures (e.g., [Quinto 1983], [Hertle 1984], [Finch 1985]). Support theorems are useful in partial differential equations [Helgason 1973] and can have implications in tomography [Shepp and Kruskal]. However, there are examples, depending on the function class (e.g., [Shepp and Kruskal]) or measure [Boman 1985], for which support theorems do not hold.

Our goal is to prove support theorems for Radon transforms with positive real analytic measures on hyperplanes in $\mathbb{R}^n$. We will use the theory of analytic pseudodifferential operators and a lovely theorem of Kawai–Kashiwara–Hörmander about analytic wave-front sets. Even for the classical transform, our

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