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A BOUNDARY VALUE PROBLEM FOR DISCRETE-VELOCITY MODELS

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1. Introduction. In spite of the fact that the Boltzmann equation has been studied for more than a hundred years, no completely satisfactory theory of existence for the equation is yet known. A survey of results concerning initial-value problems for both homogeneous and inhomogeneous equations can be found in [11]. More recently, research has continued with partial success; we mention in particular Arkeryd's [2] very general existence results (for which, however, there is no uniqueness) and the theorem of Illner and Shinbrot [14; see also 19] for a rarefied cloud of gas expanding into an infinite vacuum.

The situation is still less satisfactory for boundary value problems, even when they are steady. The linear theory was worked out twenty years ago [5–8] and work continues up to the present [3, 12, 13], but the only known nonlinear results refer to small deviations from equilibrium [9, 16, 23]. In this paper, we consider two nonlinear boundary value problems concerning steady, one-dimensional flow between parallel plates. In one, the boundaries emit particles into the region occupied by the fluid with given distributions; any particle hitting the boundary is absorbed. The physical situation being modelled here is flow between plates at different temperatures, with net evaporation from one wall and a net condensation on the other [1, 4, 17, 22].

The other problem is perhaps even more interesting. Again we consider a steady, one-dimensional flow between parallel plates, but here the boundaries emit particles with a distribution that is given modulo a factor that is used to ensure that no net flow through the boundaries occurs. This is the boundary condition usually used for such problems as the description of heat transfer between parallel plates [7, 8].

Both problems exhibit a certain similarity to the spatially homogeneous initial-value problem for the Boltzmann equation, since the equations are ordinary differential equations, although boundary problems are intrinsically more difficult. We note, however, two specific differences between the problems:

- In the boundary value problems, the derivative appearing in the equations is multiplied by the velocity of a molecule. This gives rise to certain difficulties when the velocity is zero.
- The boundary values being assigned only for incoming values of the velocity, there is no way to compute the values of the conserved moments.

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