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THE JACOBIAN ALGEBRA OF A GRADED GORENSTEIN SINGULARITY

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Introduction. The Jacobian algebra of an isolated hypersurface singularity (defined by f in the power series ring $P = \mathbb{C}\{z_1, \ldots, z_n\}$) is the algebra $P/(f_{z_1}, \ldots, f_{z_n})$. In case f is weighted homogeneous (so that $A = P/(f)$ is the completion of a graded ring), this algebra $J(A)$ is a module over A. It is isomorphic to the module T^1 of first-order deformations of A, and its length μ is an important invariant of the singularity. $J(A)$, being a complete intersection, is Gorenstein, so local duality implies it is a self-dual finite-length A-module (i.e., isomorphic to the A-module $\mathrm{Hom}_{\mathbb{C}}(J(A), \mathbb{C})$). It is also isomorphic to another geometrically important module of the dualizing differentials modulo the Kähler (or regular) differentials; we denote this module $\omega_A/\mathrm{Im}(\Omega_A^n)$.

In 1980, James Damon pointed out to us that for certain two-dimensional graded complete intersections A, the dual of T^1 is a cyclic A-module, but this is false in general in other dimensions. We then proved the following

THEOREM (2.2) below. *Let A be a graded Gorenstein isolated singularity of dimension 2. Then the dual of T^1 is a non-0 cyclic A-module.*

Our proof used the geometric description due to H. Pinkham of the graded pieces of A, with the duality assertion resulting from Serre duality on a curve. An informal research announcement entitled "T^1-duality for graded Gorenstein surface singularities" (March 1981) was circulated describing the steps (see also [23], §4). One consequence is that every quasi-homogeneous Gorenstein surface singularity has a canonical one-dimensional highest-weight deformation; in the hypersurface case, this is given by $f + tH(f)$, where $H(f)$ is the Hessian. Damon used this result to study topological triviality of the semi-universal deformation of certain A along this special subspace described by the one-dimensional highest-weight piece of T^1 [3]. We give here a different proof; the original (more difficult) one computes the graded pieces of T^1 for any normal graded A in dimension 2, and will be given in [24].

Suppose A is Gorenstein (with isolated singularity, and of positive dimension n). We define the Jacobian algebra $J(A)$ to be the cyclic A-module (hence A-algebra) defined as the dualizing module modulo the image of the Kähler n-forms. If $n > 1$, $J(A)$ is the local cohomology module $H^1_{\{m\}}(\Omega_A^n)$, where m denotes the maximal ideal. In the hypersurface case, this does not agree with the

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