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ON HILBERT MODULAR FORMS OF HALF-INTEGRAL WEIGHT

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There are two main themes in this paper: (i) the relation between the Fourier coefficients of a Hilbert modular form of half-integral weight and those of a form of integral weight; (ii) the arithmeticity of critical values of a zeta function attached to two forms of half-integral weight. Our results will generalize those in the elliptic modular case obtained in our previous papers. To describe them more explicitly, let F, $\mathfrak{o}$, $\mathfrak{d}$, $\mathbf{a}$, and $\mathbf{f}$ denote throughout the paper a totally real algebraic number field of finite degree, the maximal order of F, the different of F over $\mathbf{Q}$, the set of archimedean primes of F, and the set of nonarchimedean primes of F, respectively. We put $G = \mathrm{SL}_2(F)$ and let G act on $\mathscr{H}^{\mathbf{a}}$ as usual, where

$$\mathscr{H} = \{ z \in \mathbf{C} | \mathrm{Im}(z) > 0 \}.$$

For two fractional ideals $\mathfrak{x}$ and $\mathfrak{y}$ of F such that $\mathfrak{x}\mathfrak{y} \subset \mathfrak{o}$, we put

$$\Gamma[\mathfrak{x}, \mathfrak{y}] = \{ \gamma \in G | a_\gamma \in \mathfrak{o},\ b_\gamma \in \mathfrak{x},\ c_\gamma \in \mathfrak{y},\ d_\gamma \in \mathfrak{o} \},$$

where a_γ, b_γ, c_γ, and d_γ are the entries of γ in the standard order. By a *half-integral weight* we mean an element k of $(1/2)\mathbf{Z}^{\mathbf{a}}$ such that $2k_v$ is odd for all $v \in \mathbf{a}$; naturally an *integral weight* is an element of $\mathbf{Z}^{\mathbf{a}}$. For $\gamma \in G$, $z \in \mathscr{H}^{\mathbf{a}}$, and a weight k, we define a factor of automorphy J_k by

$$J_k(\gamma, z) = \begin{cases} \prod_{v \in \mathbf{a}} (c_v z_v + d_v)^{k_v} & (k \in \mathbf{Z}^{\mathbf{a}}), \\ h(\gamma, z) \prod_{v \in \mathbf{a}} (c_v z_v + d_v)^{k_v - (1/2)} & (k \notin \mathbf{Z}^{\mathbf{a}}), \end{cases}$$

where $(c, d) = (c_\gamma, d_\gamma)$, and $h(\gamma, z)$ is a factor of weight $1/2$ introduced in [S8]. It should be noted that $h(\gamma, z)$ is defined only for γ in a certain subset of G, but at least for $\gamma \in \Gamma[2\mathfrak{d}^{-1}, 2\mathfrak{d}]$. Then we denote by $\mathscr{M}_k$ the set of all holomorphic modular forms on $\mathscr{H}^{\mathbf{a}}$ of weight k with respect to congruence subgroups of G, defined as usual relative to J_k.

Let us now assume k to be half-integral and put $m_v = k_v - (1/2)$ for $v \in \mathbf{a}$. In parallel to the elliptic modular case, we choose a "level" which is an integral

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