ASYMPTOTICS FOR CLOSED GEODESICS IN A HOMOLOGY CLASS, THE FINITE VOLUME CASE

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1. Introduction. In this paper we will study the asymptotic behavior of a family of counting functions connected with the fundamental group Γ of a hyperbolic manifold M. If $\gamma \in \Gamma$, then in the free homotopy class of γ, $\{\gamma\}$ there is a unique closed geodesic on M; we will denote its length by ℓ_γ. It is classical that $H_1(M; Z) = \Gamma/[\Gamma, \Gamma]$ and therefore each free homotopy class $\{\gamma\}$ has a well-defined image in $H_1(M; Z)$. We will denote it by $[\gamma]$. For each $\alpha \in H_1(M; Z)$ define

$$N_\alpha(\lambda) = \#\{\{\gamma\}: \ell_\gamma < \lambda \text{ and } [\gamma] = \alpha\}. \tag{1.1}$$

We will obtain asymptotic formulae for these counting functions. A form of this problem was considered in [Ep] for hyperbolic 3-manifolds that fiber over the circle. In that work a connection between the spectral theory of the Laplace operator acting on flat line bundles and counting functions analogous to $N_\alpha(\lambda)$ was explored. In this paper we will follow the same general procedures. More recently, in [Ph–Sa1] and [Ad–Su] such formulae are obtained for the case of compact hyperbolic manifolds. The answer in the case of surfaces of genus g is

$$N_\alpha(\lambda) \sim \frac{(g-1)^g}{2}\frac{e^\lambda}{\lambda^{g+1}}. \tag{1.2}$$

We will treat the case of noncompact manifolds of finite volume. The answer in these cases depends in a more essential way on the dimension of the manifold. If M is a surface of genus g with $(p+1)$ punctures, then

$$N_\alpha(\lambda) \sim \binom{2p}{p}\frac{(2g-2+p)^{p+g}e^\lambda}{\lambda^{g+1+p}} \cdot \frac{1}{2^{g+2}}. \tag{1.3}$$

In four or more dimensions we obtain

$$N_\alpha(\lambda) \sim C_1(n, r) \cdot C_2(M)\frac{e^{(n-1)\lambda}}{\lambda^{(r/2+1)}}, \tag{1.4}$$

where $r = \dim H^1(M; R)$.

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