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CYCLE CLASSES AND RIEMANN–ROCH FOR CRYSTALLINE COHOMOLOGY

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Introduction. In this paper we show that crystalline cohomology is a Weil cohomology in the strong sense. Specifically, we prove the following theorem, where, for X a smooth variety, $A^*(X)$ is the Chow ring of algebraic cycles modulo rational equivalence:

THEOREM. *Let k be a field of characteristic $p > 0$, and fix a Cohen ring C for k. There is a natural transformation of contravariant functors from the category of proper, smooth X over k to the category of commutative graded rings with unit,*

$$\eta\colon A^*(X) \to H^{2*}(X)$$

where $H^*(X) = H^*_{\text{crys}}(X/C) \otimes_{\mathbf{Z}} \mathbf{Q}$, such that if $f\colon X \to Y$ is a projective morphism, then for all $z \in A^*(X)$,

$$f_*(\eta_X(z)) = \eta_Y(f_*(z)).$$

It follows immediately by [K1] 1.2.1 that the homomorphisms η_X factor through the groups of cycles modulo *algebraic* equivalence.

In order to understand what this theorem means, recall ([K1] 1.2) that a Weil cohomology theory consists of a contravariant functor $X \mapsto H^*(X)$ from the category of (in our case) projective smooth schemes over a field k to the category of augmented, finite dimensional, graded anticommutative algebras over a field F of characteristic zero, satisfying the following axioms:

A. *Poincaré duality.* If $\dim X = n$, then

(i) $H^i(X) = 0$ for $i < 0$ or $i > 2n$,

(ii) There is a trace map tr: $H^{2n}(X) \to F$ which is an isomorphism if X is geometrically connected,

(iii) The canonical pairings, induced by multiplication and the trace map,

$$H^i(X) \times H^{2n-i}(X) \to F$$

are perfect if X is geometrically connected.

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