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ON NONSTATIONARY FLOWS OF VISCOUS AND IDEAL FLUIDS IN $L_s^p(\mathbb{R}^2)$

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Introduction. In [10] the authors have shown that the Euler equation for incompressible fluids in $\mathbb{R}^2$ is globally well posed in any (vector-valued) Lebesgue spaces,

$$L_s^p = (1-\Delta)^{-s/2} L^p(\mathbb{R}^2), \quad \text{with } s > 1 + \frac{2}{p}, \quad \text{and} \quad 1 < p < \infty,$$

and that the same result holds for the Navier–Stokes equation uniformly in the viscosity ν. The notation L_s^p will be used indiscriminately for scalar and vector-valued functions.

The notion of well-posedness in [10] includes existence, uniqueness, and persistence property. The last means that the state $u(t)$ of the fluid at time t belongs to the same function space X as does the initial state, and describes a continuous curve in X. However, the continuous dependence on the initial data, i.e., the continuity of the map $u(0) \to u(t)$ from L_s^p into itself was not considered there.

As it was pointed out in [6], this notion of well-posedness is rather strong and is not always available in the published literature.

In the present paper we are mainly interested in proving the continuous dependence on the initial data, and the convergence with vanishing viscosity of Navier–Stokes flow to ideal flow.

The initial value problem (I.V.P.) for the Navier–Stokes equation ($\nu > 0$), and for the Euler equation ($\nu = 0$) in $\mathbb{R}^2$ may be written

$$\begin{cases} \partial_t u_j + (u \cdot \nabla) u_j = \nu \Delta u_j - \partial_j \pi & x \in \mathbb{R}^2, t > 0 \\ \operatorname{div} u = 0 \\ u(x,0) = u_0(x) \end{cases} \tag{I.1}$$

where $\partial_t = \partial/\partial t$, $\nabla = \text{grad} = (\partial_1, \partial_2) = (\partial/\partial x_1, \partial/\partial x_2)$; $u = u(x,t) = (u_1(x,t), u_2(x,t))$ is the velocity field; $\pi = \pi(x,t)$ is the pressure, $u \cdot \nabla = u_i \cdot \partial_i$ (with summation convention); and $\nu \geqslant 0$ is the kinematic viscosity ($\nu = 0$ corresponds to the case of ideal flows).

In these equations the pressure π is automatically determined (up to a function of t) if u is known; indeed $\partial \pi = -(1-P)(u \cdot \nabla)u$, where P is formally given

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