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REFINED CONJECTURES OF THE "BIRCH AND SWINNERTON-DYER TYPE"

B. MAZUR and J. TATE

To Yuri Manin on the occasion of his fiftieth birthday

The idea behind the present article is that, in certain instances, arithmetic conjectures concerning the special values of derivatives of p-adic L functions can be "refined" to obtain formulations of *stronger* conjectures. These stronger conjectures *avoid any mention of p-adic L functions* and therefore obviate the necessity of constructing the p-adic L functions for the statement of the conjectures. Moreover, they avoid any reference to a prime number p, and require no p-adic limiting process; they should ultimately be phrased, perhaps, in adelic language. In this paper, however, we state our conjectures "at a finite layer M", where M is a possibly composite number (somewhat restricted). Even when $M = p$, however, our conjecture "at layer p" is not implied by the analogous conjecture for the p-adic L function. Indeed, our conjecture predicts congruence formulas modulo divisors of $p - 1$, in this case. When M is a product of distinct primes, our conjectured congruence formulas involve what seems to us to be a thoroughgoing mixture of phenomena related to those prime divisors.

In 1974 ([Man]) Yuri Manin expressed the hope that there should exist "functions with adelic type domains of definition and ranges of values" for which p-adic L functions are only a component. We are pleased, on the occasion of Manin's fiftieth birthday, to be able to offer, at least, a very fragmentary piece of "experimental mathematics" which is resonant with Manin's hope.

Here is the specific context in which we work: Let $A_{/\mathbf{Q}}$ be an elliptic curve admitting a modular parametrization,

$$f\colon X_0(N)_{/\mathbf{Q}} \to A_{/\mathbf{Q}}.$$

In [M-T-T], p-adic analogues of the conjectures of Birch and Swinnerton-Dyer for $A_{/\mathbf{Q}}$ were formulated, in the case when p has ordinary reduction for A. Numerical evidence was gathered in support of these conjectures. The conjectures of [M-T-T] are formulas, whose *left hand sides* are special values of derivatives of the p-adic L function $L_p(A, s)$ and whose *right hand sides* are expressions involving arithmetic invariants of $A_{/\mathbf{Q}}$ among which is a "regulator term", or, as it is referred to in [M-T-T], the "*sparsity*". In the case where p is a prime of split multiplicative reduction for A, the "regulator term" involves the p-adic logarithm

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