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NOTES ON MOTIVIC COHOMOLOGY

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Dedicated to Yu. I. Manin on the occasion of his 50th birthday

Introduction. Imagine a world in which the K-theory $K^*(X)$ of a topological space X had been defined, but the ordinary cohomology groups $H^*(X)$ had not yet been discovered. Then the rational cohomology groups $H^*(X, \mathbf{Q})$ would be known at least as functors, since they could be defined as the associated graded of the Atiyah–Hirzebruch filtration of $K^*(X) \otimes \mathbf{Q}$. However, there would be at least three difficulties in the situation:

1. This world would not have our explicit cocycles representing cohomology classes. These are often interesting geometric objects like differential forms or Cech cocycles which relate cohomology to other mathematical ideas.
2. The integral cohomology groups would not be defined.
3. The powerful computational techniques of cohomology theory would not be available.

In such circumstances, one might well expect to find a quest going on for a geometrically defined cohomology theory which, when tensored with the rationals, would admit a Chern character isomorphism from $K^*(X) \otimes \mathbf{Q}$.

Present day algebraic geometry is such a world. The algebraic K groups $K_i(X)$ of an algebraic variety X have been known since 1973 [Q], but a cohomology theory which is appropriately related to these algebraic K-groups has not been found. Precise conjectures as to what properties this hoped-for cohomology theory should have were given in [B2] for the Zariski topology, and then in [L1] for the étale topology. In a sense the quest for the hoped-for cohomology theory dates back to Grothendieck who, working before algebraic K-theory was defined, was looking for a cohomology theory of algebraic varieties with certain universal properties. (We know about Grothendieck's ideas through the work of Manin [M].) Following Grothendieck, we call the hoped-for cohomology theory *motivic cohomology*. (Lichtenbaum calls it arithmetic cohomology in honor of its conjectured relations with number theory [B1], [B2], [L1], [L2], which constitute one of the main reasons for seeking it.)

In this introduction, we give some of the most basic expected properties of motivic cohomology, based on its conjectured connection with algebraic K-theory. For a more detailed account, see [B2].

0.1 *The γ-filtration.* The analogue for algebraic K-theory of the Atiyah-Hirzebruch filtration on topological K-theory is the γ-filtration [So]. Consider the

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