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R-EQUIVALENCE ON CONIC BUNDLES OF DEGREE 4

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To Yu. I. Manin on his fiftieth birthday

0. Introduction. Let k be a field of characteristic zero, $\bar{k}$ an algebraic closure of k, and $\mathfrak{g} = \mathrm{Gal}(\bar{k}/k)$. Let X/k be a smooth projective surface which admits a dominant k-morphism p: $X \to \mathbb{P}_k^1$ with smooth generic fibre of genus zero and with 4 geometric degenerate fibres, and assume that p is relatively minimal. Then any geometric degenerate fibre is the transverse intersection of two exceptional curves of the first kind ([10], 1.6) and the degree $(\omega_X \cdot \omega_X)$ of X is 4. According to Iskovskikh [7], [8] (see also [9], §3), two cases may occur: either the anticanonical bundle ω_X^{-1} is not ample, in which case X is a generalized Châtelet surface as soon as $X(k) \neq \varnothing$ (see [9]), or ω_X^{-1} is ample, in which case X is a k-minimal Del Pezzo surface of degree 4 with Pic X free of rank 2 (see [8] and [9]).

These last surfaces, as soon as the set $X(k)$ of their rational points is not empty, may also be characterized [9] as those smooth complete intersections of two quadrics $X \subset \mathbb{P}_k^4$ which are given in homogeneous coordinates by a system of two quadratic forms with coefficients in k of the following type:

$$\begin{cases} f(x_0, \ldots, x_4) = 0, \\ x_1 x_2 + x_3 x_4 = 0. \end{cases}$$

In Manin's description of the possible actions of $\mathfrak{g}$ on the 16 lines of a Del Pezzo surface of degree 4 ([10]; [12], 2nd edition, p. 178), they correspond to those actions such that no orbit crosses the middle vertical line in Table 2 (see [8] and [9]).

In descent theory over a given smooth proper rational k-surface X [3], [4], which was developed after work of F. Châtelet [1] and Yu. I. Manin ([12], chap. VI), J.-J. Sansuc and the first named author have raised two basic questions, an affirmative answer to which would solve most natural arithmetico-geometric problems concerning X:

(Q1) *If $\mathcal{T}$ is a universal torsor over X with $\mathcal{T}(k) \neq \varnothing$, is $\mathcal{T}$ a k-rational variety?*

(Q2) *If k is a number field and $\mathcal{T}$ a universal torsor over X, does $\mathcal{T}$ satisfy the Hasse principle?*

In [6], an affirmative answer was given to both questions when X is (a suitable model of) a generalized Châtelet surface, i.e. when X belongs to the first

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