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SELECTED PROBLEMS OF SUPERMANIFOLD THEORY

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For Yu. I. Manin's fiftieth birthday

> ... being at times somewhat of a humorist myself I know that it is hard to have an amusing story to tell and find no listener.
> S. Maugham.

Supermanifold theory is very popular nowadays. Its fantastic applications in physics (unified theory of all the known forces) and some not so spectacular results in mathematics, discussed and reviewed in thousands of papers and dozens of books, screen somewhat its shortcomings, the most tantalizing mysteries.

The best introductions to supermanifold theory that give you its specific character are at present [1–3]; see also [4–6] and references therein. The most extensive exposition of supermanifold theory (calculus, differential geometry, representation theory, mechanics, quantization, etc.) is contained in the workouts of my "Seminar on Supermanifolds" (1977–86), which while in preparation to be published by D. Reidel are preprinted by Stockholm University. The vague hints at some results that the reader will find in what follows refer to "Seminar."

I will start with the most exciting problems. They also seem to be rather difficult. Simpler ones, to keep you busy for a couple of evenings, are contained in "Seminar"'s Problems.

(1) *Generalize the notion of supermanifold.* (a) Supergroups are better than groups because they provide us with a bigger stock of automorphisms of the same entities, which is the whole point. In addition, it turns out that by introducing nilpotents and noncommutative algebras instead of algebras of functions in this particular way (via supermanifolds and their rings of functions), we are able to retain all the notions of differential geometry! And even more so, on supermanifolds practically all of them have at least two versions.

But after introducing such a nice generalization of groups, we suddenly confine ourselves only to automorphisms of *homogeneous* objects, and those automorphisms themselves are only *even*. This does not seem fair, and from the mathematical point of view looks arbitrary. Physicists may refer to Pauli's principle on the relation of spin and statistics, but is it not an artifact of the constraints

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