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CYCLIC HOMOLOGY OF DIFFERENTIAL OPERATORS

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To Dear Yuri Ivanovich on His Fiftieth Birthday

1. Let $\mathscr{D}(X)$ denote the $\mathscr{k}$-algebra of differential operators on a smooth manifold X in one of the following categories: algebraic, holomorphic or C^∞. In the first case X has to be an affine variety over the ground field $\mathscr{k}$ of characteristic zero, in the second case a Stein manifold ($\mathscr{k} = \mathbb{C}$), assumed, for simplicity, to possess finitely many connected components, and in the last case a compact C^∞-manifold (possibly with boundary or nonorientable; $\mathscr{k} = \mathbb{R}$ or $\mathbb{C}$). The purpose of this article is to determine Hochschild and cyclic homology of $\mathscr{D}(X)$ denoted, respectively, $H_*(\mathscr{D}(X), \mathscr{D}(X))$ and $HC_*(\mathscr{D}(X))$. In the holomorphic and C^∞ settings, $\mathscr{D}(X)$ is naturally a locally convex algebra with respect to $\hat{\otimes}_\pi$-tensor product, and the groups above mean the corresponding *topological* homology groups. For basic definitions and properties of cyclic homology see [5] and for basics on locally convex homological algebra consult [4] and [7].

2. THEOREM.

$$H_q(\mathscr{D}(X), \mathscr{D}(X)) \simeq H_{\mathrm{DR}}^{2n-q}(X) \qquad (q \in \mathbb{N};\ n = \dim X). \tag{1}$$

3. THEOREM.

$$HC_q(\mathscr{D}(X)) \simeq H_{\mathrm{DR}}^{2n-q}(X) \oplus H_{\mathrm{DR}}^{2n-q+2}(X) \oplus H_{\mathrm{DR}}^{2n-q+4}(X) \oplus \cdots \qquad (q \in \mathbb{N}). \tag{2}$$

4. Remark. In proof of the holomorphic case of Theorem 3 we shall assume, for simplicity, that $H_{\mathrm{DR}}^*(X)$ is finite-dimensional; the similar condition automatically holds in the two remaining cases.

The isomorphisms in (1) are canonical and functorial with respect to embeddings of codimension zero. The proof of Theorem 3 which is presented below will provide similarly functorial isomorphisms in (2), for $q \geqslant 2n-1$. The existence of *canonical* isomorphisms in the "unstable" range $q < 2n-1$ can be proved as well, at least in C^∞ case, but requires stronger means (cf. Remarks 8 and 13.1 below).

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