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RAMIFIED TORSION POINTS ON CURVES

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Dedicated to Yu I. Manin on the occasion of his fiftieth birthday

Introduction. Let K be a field and C a smooth complete connected curve over K. Let $\bar{K}$ denote an algebraic closure of K. If $P, Q \in C(\bar{K})$ we write $P \sim Q$ if a positive integral multiple of the divisor $P - Q$ is principal. Then " $\sim$ " is an equivalence relation. With respect to this relation, we call an equivalence class a torsion packet.

Using Abel's addition theorem, the Manin-Mumford conjecture proven by Raynaud [R-2] is equivalent to:

THEOREM A. *If* $\mathrm{char}(K) = 0$ *and the genus of C is at least two then every torsion packet in* $C(\bar{K})$ *is finite.*

We propose the following conjecture:

CONJECTURE B. *Suppose K is a number field and T is a torsion packet in* $C(\bar{K})$ *stable under* $\mathrm{Gal}(\bar{K}/K)$. *Suppose* $\mathfrak{p}$ *is a prime of K satisfying*

(i) char $\mathfrak{p} > 3$.

(ii) $K/\mathbb{Q}$ *has good reduction at* $\mathfrak{p}$.

(iii) C *has good reduction at* $\mathfrak{p}$.

If the genus of C is at least two, the extension $K(T)/K$ *is unramified above* $\mathfrak{p}$.

We have proven this conjecture under any of the following additional hypotheses:

(a) char $\mathfrak{p} > 2g$.

(b) C has ordinary reduction at $\mathfrak{p}$ (i.e., the Hasse-Witt matrix at $\mathfrak{p}$ is invertible).

(c) C has superspecial reduction at $\mathfrak{p}$ (i.e., the Hasse-Witt matrix at $\mathfrak{p}$ is zero).

(d) C is an abelian branched covering of $\mathbb{P}^1_K$ over K unbranched outside $\{0, 1, \infty\}$, and T is the torsion packet on C containing the inverse image of $\{0, 1, \infty\}$.

In this paper we will prove the conjecture under any of the hypotheses (a)–(c). This generalizes the first part of Theorem A of [C-1]. In [C-2] we will prove the conjecture under hypothesis (d). (See also [C-3] for additional evidence.) We will also give a new proof, of the Manin-Mumford conjecture, which we will now sketch.

First, by standard arguments it suffices to prove Theorem A when K is a number field. Second, Bogomolov [B-1] has proven.

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