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QUANTUM FIELD THEORIES IN ONE AND TWO DIMENSIONS

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1. Introduction. All known physical forces with the exception of gravity have been described by quantum field theories. This is probably the most important achievement of theoretical physics in the past fifty years. However, for several decades this success led to an estrangement between physics and mathematics, as we have not yet learnt to analyse those complex theories with methods which are both efficient and rigorous. In recent years, however, physicists found uses for simpler systems of that kind, mainly as tools for the description of surface phenomena in statistical physics and in attempts to understand the relation of gravity to the other forces of nature, and mathematicians discovered that these systems could be adapted to their purposes. A striking discovery along these lines was the realization that one of the simplest possible quantum field theories has the biggest sporadic simple group F_1 (or FG) as automorphism group.

Good introductions to quantum field theory are available (Streater and Wightman 1978, with further references), but they are mostly geared towards the treatment of more complex systems. Here I will give an introduction to the simplest quantum field theories and some new results. I hope that this will contribute to the new upsurge of interactions between the mathematics and physics communities, to which Yu. I. Manin has contributed so much.

Section 2 introduces a set of rather general quantum field theories, which should include the experimentally relevant ones in four dimensions. However, features not used in sections 3, 4 are not discussed. The main concern is the dimension of local fields, for which I found no sufficient discussion elsewhere. In section 3 several important examples in one dimension are discussed, and section 4 gives some new results for particularly simple chiral theories in two dimensions. The examples in section 3 include the one with symmetry group F_1. The results of section 4 apparently are related to the correspondence between the ADE series of simple Lie algebras and the finite subgroups of SU(2).

2. Axioms for quantum field theories. Quantum field theories model the causal structure of space-time, the states of a physical system, the results of measurements performed on the states, and the interrelationships between these structures.

Space-time is modelled by a differentiable manifold M of dimension d equipped with a causal structure.

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