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A NEW INTERPRETATION OF GELFAND–TZETLIN BASES

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To Yuri Ivanovich Manin on his 50th birthday

Introduction. In this paper we determine q-analogues of Gelfand-Tzetlin bases for a q-deformation $\mathscr{D}_N^q$ of the universal enveloping algebra $\mathscr{U}(\widetilde{\mathfrak{gl}}_N)$ for the Lie algebra $\widetilde{\mathfrak{gl}}_N = \mathfrak{gl}_N \otimes_{\mathbb{C}} \mathbb{C}[[x]]$. The definition of $\mathscr{D}_N^q$ (and its generalization) was given by V. G. Drinfeld and M. Jimbo (see [1, 2]). We introduce $\mathscr{D}_N^q$ (§1) in a somewhat different way by means of R-matrix technique, developed by L. D. Faddeev and his collaborators (see e.g. [3]). This technique was applied to construct first some "elliptic" deformations of $\mathscr{U}(\mathfrak{gl}_2)$ [4], $\mathscr{U}(\mathfrak{gl}_N)$ [5] and then to introduce the "elliptic" R-algebras of level n [6]. The latter are deformations of $\widetilde{\mathfrak{gl}}_N \mathrm{mod}(x^n)$ and also natural generalizations of $\mathscr{D}_N^q$ as $n \to \infty$.

We use the results and methods of [6, 7] in this paper but can prove our "branching" theorems only in the special case of $\mathscr{D}_N^q$ since the branching properties for the elliptic R-algebras are of a more complicated nature than for $\mathscr{D}_N^q$. These properties are analoques of the well-known Young–Gelfand–Tzetlin rules for decomposing irreducible finite-dimensional representations of $\mathfrak{gl}_N$ or S_n under the action of $\mathfrak{gl}_K \subset \mathfrak{gl}_N$ or $S_k \subset S_n$ and are most important in our paper. It was shown recently by G. I. Olshansky that the Yangians, which were introduced by V. G. Drinfeld and are the limits of $\mathscr{D}_N^q$ as $q \to 1$, play an essential role in calculating the centralizers of $\mathscr{U}(\mathfrak{gl}_K)$ in $\mathscr{U}(\mathfrak{gl}_N)$. For S_n the degenerated Hecke algebras from [8] arise when we find the centralizers of $\mathbb{C}[S_k]$ in $\mathbb{C}[S_n]$ (cf. [6, 9]). We develop these results, generalizing the classic branching properties and extending them to $\mathscr{D}_N^q$ and affine Hecke algebras in §2, 3.

We apply the structural theorems for $\mathscr{D}_N^q$ obtained in this way to their irreducible representations, connected with the so-called skew Young diagrams. This class includes q-analogues of finite-dimensional irreducible representations of $\mathfrak{gl}_N$, constructed independently in [10, 11] for $\mathscr{D}_N^q$ of level = 1 and in [6, 7] for its elliptic generalization. As we prove (for sufficiently large N) at the end of the paper, only representations of the above type have the classic branching properties. We do not detail here the structure of the considered representations (e.g., do not describe the action of generators of $\mathscr{D}_N^q$ on Gelfand–Tzetlin bases). For further information we refer to [11] (where the latter problem was solved for level = 1) and the author's paper on the second Weyl character formula to be published soon (see also [6, 9] for Hecke algebras).

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