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TORSION POINTS ON FERMAT JACOBIANS, ROOTS OF CIRCULAR UNITS AND RELATIVE SINGULAR HOMOLOGY

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Dedicated to Yu. I. Manin on the occasion of his fiftieth birthday

Introduction. Our object is to analyze certain relationships among galois modules arising as relative singular homology groups. The theory developed in this paper permits one to prove, for example, the following result: Let N be a positive integer. Let U denote the affine Fermat curve

$$x^N + y^N = 1$$

over $\mathbb{Q}$. Let $\overline{U}$ denote the projective closure of U. Let ∞ denote the squarefree effective divisor on $\overline{U}$ of support the complement of U in $\overline{U}$. Let S denote the generalized jacobian of $\overline{U}$ of conductor ∞. Then S is an extension of an abelian variety by a torus and is defined over $\mathbb{Q}$. Let b denote the $\mathbb{Q}$-rational point of S corresponding to the difference of the points $(0, 1)$ and $(1, 0)$ of U.

THEOREM 0. *The numberfield generated by the coordinates of the N^{th} roots of the point b in the group $S(\overline{\mathbb{Q}})$ contains the numberfield generated by the roots of the equation*

$$1 - (1 - x^N)^N = 0.$$

If N is prime, the asserted containment becomes an equality.

A similar result was obtained by Greenberg [G].

The paper consists of two parts. Let X be a smooth quasi-projective $\mathbb{Q}$-scheme, D an effective divisor on X with singularities no worse than normal crossings and N a positive integer. The purpose of Part I is to explain how to equip relative singular homology groups of the form $H_*(X(\mathbb{C}), D(\mathbb{C}); \mathbb{Z}/N\mathbb{Z})$ with a natural action of

$$G(\mathbb{Q}) \stackrel{\text{def}}{=} \text{the galois group over } \mathbb{Q} \text{ of the algebraic closure of } \mathbb{Q} \text{ in } \mathbb{C}.$$

We employ the methods of étale homotopy theory [AM] to do this. There is no novelty to be claimed for Part I other than for the approach, which is to proceed

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