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# DE RHAM COHOMOLOGY AND CONDUCTORS OF CURVES

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An *arithmetic surface* $\mathscr{X}$ is a regular scheme $\mathscr{X}$ which is flat and proper over Spec(Z) with fibre dimension 1. The zeta function of $\mathscr{X}$, $\zeta(\mathscr{X}, s)$, is defined by

$$\zeta(\mathscr{X}, s) = \prod_{\substack{x \in \mathscr{X} \\ \text{clsd. pt.}}} \left(1 - N(x)^{-s}\right)^{-1}$$

where $N(x)$ denotes the number of elements in the residue field at the closed point $x$ [7]. One conjectures that $\zeta(\mathscr{X}, s)$, which is known to be analytic in a half plane, extends meromorphically to C and satisfies a functional equation. More precisely, one is *given* in addition to $\zeta(\mathscr{X}, s)$ a "$\Gamma$-factor" $\Gamma(\mathscr{X}, s)$ and a positive rational number $A$, *the conductor*, and one conjectures [6] that

$$\xi(\mathscr{X}, s) = A^{s/2}\zeta(\mathscr{X}, s)\Gamma(\mathscr{X}, s)$$

admits a meromorphic extension and satisfies

$$\xi(\mathscr{X}, s) = \pm\xi(\mathscr{X}, 2 - s).$$

The purpose of this note is to relate the conductor $A = A(\mathscr{X})$ to the complex of absolute de Rham differentials

$$\Omega^{\cdot}_{\mathscr{X}} = \Omega^{\cdot}_{\mathscr{X}/\mathbf{Z}} = \left\{ \mathcal{O}_{\mathscr{X}} \to \Omega^1_{\mathscr{X}/\mathbf{Z}} \to \Omega^2_{\mathscr{X}/\mathbf{Z}} \right\}.$$

Let $C^{\cdot}$ be a bounded complex of abelian sheaves on a scheme $\mathscr{X}$ proper over Spec(Z). Assume the $C^i$ are Z-torsion and coherent sheaves. It follows that the hypercohomology $\mathrm{H}^i(\mathscr{X}, C^{\cdot})$ are finite groups. We write

$$\mathrm{X}(C^{\cdot}) = \prod_i \left(\#\mathrm{H}^i(\mathscr{X}, C^{\cdot})\right)^{(-1)^i}$$

for the (multiplicative) Euler characteristic. Our first result was conjectured by K. Kato and proved by him in the equicharacteristic case for $\mathscr{X}$ of arbitrary dimension [4].

THEOREM 1. *Let $\mathscr{X}$ be an arithmetic surface, and let $\Omega^{\cdot}_{\mathscr{X},\text{tors}} \subset \Omega^{\cdot}_{\mathscr{X}}$ be the subcomplex of* Z*-torsion differentials. Then the conductor $A(\mathscr{X}) = \mathrm{X}(\Omega^{\cdot}_{\mathscr{X},\text{tors}})$.*

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