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EXCEPTIONAL VECTOR BUNDLES ON PROJECTIVE SPACES

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In this article we construct an infinite set of exceptional vector bundles on $\mathbb{P}^n$. We prove that all exceptional bundles on $\mathbb{P}^2$ are constructed in this way and their dimensions are Markov numbers. As a by-product we get some spectral sequences that are generalizations of the Beilinson spectral sequences.

In the paper we shall study algebraic coherent sheaves on $\mathbb{P}^n$. We shall call such a sheaf F an exceptional sheaf iff

$$\dim \operatorname{Ext}^0(F, F) = 1, \qquad \operatorname{Ext}^i(F, F) = 0 \quad \text{where } i > 0.$$

Our ground field is a complex number field but we believe that almost all our results are valid over an arbitrary field. An exceptional sheaf F on $\mathbb{P}^n$ must be locally free and the associated vector bundle we shall also call exceptional. For $n = 2$ such bundles were studied in [2] but there they had a different definition, an exceptional sheaf means a stable sheaf with discriminant less then $1/2$. We prove that for $\mathbb{P}^2$ this definition and ours are equivalent. It is an interesting question whether an exceptional sheaf on $\mathbb{P}^n$ for $n > 2$ is stable.

The first examples of exceptional sheaves are $\mathcal{O}(i)$. It turns out that other exceptional sheaves on $\mathbb{P}^n$ are naturally composed in infinite series (E_i), $i \in \mathbb{Z}$ and we call them helixes. The series $(\mathcal{O}(i))$ is an example of helix. We define mutations of helixes and via these mutations we obtain an infinite set of helixes on $\mathbb{P}^n$. We prove that for $\mathbb{P}^2$ this set consists of all helixes.

Each helix produces two spectral sequences in the manner of the Beilinson spectral sequences for the helix $(\mathcal{O}(i))$.

We find also that for each helix on $\mathbb{P}^2$ there is a corresponding integral solution of the Markov equation $x^2 + y^2 + z^2 = 3xyz$. It is known that integral

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