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A NOTE ON PSEUDO-CM REPRESENTATIONS AND DIFFERENTIAL GALOIS GROUPS

Dedicated to Y. I. Manin on his fiftieth birthday

NICHOLAS M. KATZ AND RICHARD PINK

Introduction. This note, a sequel to [Ka-1], falls into two parts. In the first, we give a criterion for a connected semisimple algebraic subgroup of $\mathrm{GL}(n)$ to be one of the following subgroups:

$\mathrm{SL}(n)$, if $n \geqslant 2$
$\mathrm{Sp}(n)$ or $\mathrm{SO}(n)$, if n is even and $\geqslant 4$.

The criterion is based on the classification of what we call "pseudo-CM representations", a natural generalization of the notion of "CM representation" introduced in [Ka-1]. The second part applies these results to determine the differential galois groups of some concrete differential equations, including the general Kloosterman equation on $\mathbf{G}_m$.

Part 1. Throughout this section, k is an algebraically closed field of characteristic zero, n is an integer $\geqslant 2$, V is an n-dimensional vector space over k, G is a Zariski closed subgroup of $\mathrm{GL}(V)$, and G^0 is the identity component of G.

THEOREM 1. *Suppose that*

(1) *G^0 lies in* $\mathrm{SL}(V)$.
(2) *As G^0-representation, V is irreducible.*
(3) *There exists an element g in G, and a connected torus T in G^0 such that*
   (a) *As T-representation, V is the direct sum of n distinct characters $c_1, \ldots, c_n$.*
   (b) *T is $\mathrm{Ad}(g)$-stable, i.e., $gTg^{-1} = T$.*
   (c) *The automorphism $\mathrm{Ad}(g)$ of T cyclically permutes the n characters $c_1, \ldots, c_n$.*

Then there exist

an integer $r \geqslant 1$
a factorization of n as $n = n_1 \ldots n_r$ with all $n_i \geqslant 2$ and the n_i pairwise relatively prime.
algebraic groups $G_1, \ldots, G_r$, with each G_i equal to one of the groups
   $\mathrm{SL}(n_i)$ *is odd or* $= 2$
   $\mathrm{SL}(n_i)$ *or* $\mathrm{Sp}(n_i)$ *or* $\mathrm{SO}(n_i)$ *if n_i is even* $\geqslant 4$.

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