Vol. 54, No. 1 DUKE MATHEMATICAL JOURNAL © 1987

ON THE MONODROMY GROUPS ATTACHED TO CERTAIN FAMILIES OF EXPONENTIAL SUMS

Dedicated to Y. I. Manin on his fiftieth birthday

NICHOLAS M. KATZ

Introduction. The main theme of this paper is that very innocuous looking one-parameter families of exponential sums over finite fields can have quite strong variation as the parameter moves. Even when the parameter variety is the affine line $\mathbf{A}^1$ or the multiplicative group $\mathbf{G}_m$ over a finite field, the algebraic group G_{geom} which controls the variation is often "as large as possible". These results are the finite-field analogue of the fact that in characteristic zero, very simple differential equations on $\mathbf{A}^1$ and on $\mathbf{G}_{\mathrm{m}}$ can have very large differential galois groups.

In Part 1 we develop some general results concerning irreducible lisse sheaves on open curves in characteristic $p > 0$, in part modeled on [Ka-2] and [Ka-Pi]. In Parts 2 and 3 we calculate G_{geom} for the one-parameter families of exponential sums in characteristic p which correspond to the Kloosterman and Airy differential equations respectively of any rank $n \geqslant 2$. It is very striking that in both of these cases, our results on G_{geom} are in perfect analogy with the results of [Ka-2] and [Ka-Pi] on the differential galois group G_{gal} of the corresponding differential equation, as soon as $p > 2n + 1$. Indeed, one can speculate that in some future "motivic grand unification", the two sorts of results will both be "realizations" of a single motivic result. For the moment, we must be content to offer the results themselves as indirect evidence for the existence of such a unification.

Part 1. Lisse sheaves on open curves: general results. Throughout this paper, we fix a prime number p, an algebraically closed field k of characteristic p, a prime number $\ell \neq p$, and an algebraic closure $\overline{\mathbf{Q}}_\ell$ of $\mathbf{Q}_\ell$. Let U be a smooth connected affine curve over k, and $\mathscr{F}$ a lisse $\overline{\mathbf{Q}}_\ell$-sheaf on U of rank $n \geqslant 1$. We denote by π_1 the fundamental group $\pi_1(U, \bar{\eta})$ of U with base point a geometric generic point $\bar{\eta}$ of U, by ρ the n-dimensional $\overline{\mathbf{Q}}_\ell$-representation of π_1 on $\mathscr{F}_{\bar{\eta}}$ which $\mathscr{F}$ "is", by G_{geom} the Zariski closure of $\rho(\pi_1)$ in $\mathrm{GL}(n, \overline{\mathbf{Q}}_\ell)$, and by $(G_{\text{geom}})^0$ the identity component of G_{geom}. We say that $\mathscr{F}$ is irreducible if ρ is irreducible as a representation of π_1, or equivalently if G_{geom} acts irreducibly in its given n-dimensional representation. We say that $\mathscr{F}$, or ρ, is Lie-irreducible if the restriction of ρ to $(G_{\text{geom}})^0$ is irreducible, or equivalently if the restriction of ρ

Received October 11, 1986.