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ON THE HOMOLOGY OF POISSON ALGEBRA OF ODD VARIABLES

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To my teacher Yu. I. Manin on the occasion of his 50th birthday

Traditional algebraic K-theory is based on the group $\mathrm{GL}(\infty)$. In the case of supergroups there are two analogs of this group; $\mathrm{GL}(\infty \mid \infty)$ and $Q(\infty)$. Hence one can define two superanalogs of K-groups. They seem to complement each other; namely they seem to be the parts of different parity of bigger "super K-groups" (see, e.g. [5]). But in the supercase we have another remarkable group closely related to GL and Q. It is the group of canonical transformations in mechanics with odd variables. This group can be regarded as the classical (as opposed to quantum) version of GL and Q. Namely, this group has a nontrivial deformation; if the number of odd variables is even, it deforms into the group of GL-type and if the number is odd, it deforms into the group of Q-type. (Recall that in the case of anticommuting variables symplectic form is symmetric in the usual sense, so mechanics can be defined for any number of variables, not necessarily even, as in the case of commuting variables.) So it seems tempting to regard "the classical limit" of K-theory. This limit is very interesting. The computation of the classical additive $K_2^+(\Lambda)$ (it is sketched in §1) shows that it contains $\Omega^1(\Lambda)$. (Here Λ is a ring under investigation and $\Omega^1(\Lambda)$ is the space of differential one-forms of Λ.) This result should be compared with the "quantum" one: the usual $K_2^+(\Lambda)$ contains $\Omega^1(\Lambda)/d\Lambda$ rather than $\Omega^1(\Lambda)$ [2], [3]. It suggests the idea that the role of the classical additive Milnor group $K_i^+(\Lambda)$ is played by $\Omega^{i-1}(\Lambda)$. We show in §2 that the primitive part of the i-th stable homology group does really contain $\Omega^{i-1}(\Lambda)$ as the direct summand. This constitutes the main result of the paper. In §2.5 we make some notes about the first unstable homology group and the restriction of related cocycles to the algebra of linear canonical transformations.

Next, in §3 we investigate the stability questions. The naive embedding of the group of canonical transformations of N variables to the one of $N + 1$ variables (the embedded transformation acts only on the first N variables) does not suit us. One of the reasons is that the quantum deformations of these two groups are quite different. Another reason is that this embedding induces the zero map on the greater part of homology groups and so it cannot be used to define stable homology groups. Thus we have to look for another embedding. And really it

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