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DUALITY THEOREMS FOR NÉRON MODELS

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Introduction. In this paper we prove a local duality theorem for a Néron model over a complete discrete valuation ring with finite residue field, which generalises Tate's duality for abelian varieties over local fields. From this theorem we derive a flat duality theorem for $A[n]$, the kernel of multiplication by n on a Néron model with semistable reduction. This is similar to the local flat duality theorems of Mazur, Roberts, and Milne ([13], [15], and [18]); it is not a consequence of those theorems, however, since $A[n]$ is not necessarily a flat group scheme. We then deduce some global analogues of these theorems, culminating in an explanation in modern language of the global duality theorems of Tate [27], Cassels [6], and Bashamakov [3]. For a comprehensive account of arithmetic duality theorems, see Milne [16]. Other generalisations of Tate duality, for local fields or rings with algebraically closed residue field, were first obtained by Ogg [21] and Shafarevich [23], and later by Bégueri [4], Bester [5], and Vvedens'kii [28].

We now give some details. Let R be a complete discrete valuation ring with finite residue field. Let $S = \operatorname{Spec}(R)$ and let K be the fraction field of R. Let A_K be an abelian variety over K, and let A be its Néron model. Let $\hat{A}_K$ and A' be, respectively, the dual abelian variety and its Néron model. Let A_0 be the special fiber of A, and let A^0 be the subgroup of A whose generic fiber is A_K and whose special fiber is A_0^0, the connected component at the identity of A_0. Let $\Phi = A_0/A_0^0$, and let Φ' be the corresponding group for A'. Then Grothendieck [8] defines a pairing

$$\Phi \times \Phi' \to \mathbb{Q}/\mathbb{Z},$$

which expresses the obstruction to prolonging to S the isomorphism

$$\hat{A}_K \approx \mathscr{E}xt^1(A_K, \mathbb{G}_m)$$

on the generic fiber. Specifically, if $\Gamma \subseteq \Phi$ and $\Gamma' \subseteq \Phi'$ are subgroup-schemes which annihilate each other under Grothendieck's pairing, and if A^Γ (resp. $A'^{\Gamma'}$) is the subscheme of A (resp. A') whose generic fiber is A_K (resp. $\hat{A}_K$) and whose special fiber is the inverse image in A_0 (resp. A'_0) of Γ (resp. Γ') then there is a canonical map

$$A'^{\Gamma'} \to \mathscr{E}xt^1\big(A^\Gamma, \mathbb{G}_m\big)$$

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