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UNITARY REPRESENTATIONS OF THE VIRASORO ALGEBRA

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Introduction. The Virasoro algebra $\mathscr{L}$ is the Lie algebra over $\mathbf{C}$ of the following form:

$$\mathscr{L} = \sum_{n \in \mathbf{Z}} \mathbf{C} e_n \oplus \mathbf{C} e_0', \tag{1}$$

with the relations

$$[e_m, e_n] = (m-n) e_{m+n} + \frac{m^3 - m}{12} \delta_{m+n,0} e_0' (m, n \in \mathbf{Z}); \tag{2}$$

$$e_0' \in \text{the center of the Lie algebra } \mathscr{L}.$$

The Lie algebra of this type was first appeared in the dual string model of elementary particle physics (cf. S. Mandelstam [12]). Quite recently the Virasoro algebra was used to analyze critical phenomena in the two dimensional statistical physics (cf. A. A. Belavin–A. M. Polyakov–A. B. Zamolodchikov [1]).

Introduce the triangular decomposition $\mathscr{L} = \mathfrak{n}_+ \oplus \mathfrak{h} \oplus \mathfrak{n}_-$ of $\mathscr{L}$, where

$$\mathfrak{n}_\pm = \sum_{n \geq 1} \mathbf{C} e_{\pm n}; \qquad \mathfrak{h} = \mathbf{C} e_0 \oplus \mathbf{C} e_0'. \tag{3}$$

By V. G. Kac [8], for each $(h, c) \in \mathbf{C}^2$, there exists an irreducible $\mathscr{L}$-module $L(h, c)$, unique up to an isomorphism, with the following property. There exists a

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