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THE EQUATIONS DEFINING CHOW VARIETIES

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Introduction. Given an n-dimensional projective variety $W_n \subset \mathbf{P}_N$, the set of linear spaces of dimension $N - n - 1$ meeting W form a hypersurface in the Grassmannian $G(N - n, N + 1)$. This hypersurface or the homogeneous form in the Plücker coordinates defining it are known as the *Chow form* of W. In two remarkable papers [1], [2], Cayley defined this notation for space curves. He went on to attempt to ascertain which hypersurfaces in $G(2,4)$ arise as the Chow forms of space curves. He obtained the striking result that if F is an irreducible homogeneous polynomial in the Plücker coordinates p_{ij} then the condition

$$\frac{\partial F}{\partial p_{12}}\frac{\partial F}{\partial p_{34}} - \frac{\partial F}{\partial p_{13}}\frac{\partial F}{\partial p_{24}} + \frac{\partial F}{\partial p_{14}}\frac{\partial F}{\partial p_{23}} = 0 \tag{0.1}$$

is equivalent to the hypersurface X that F defines being either the Chow form of a space curve or the set of lines tangent to a surface in $\mathbf{P}_3$.

In §1, we derive an intrinsic geometric characterization of Chow forms. If $X \subset G(N - n, N + 1)$ is a hypersurface, and for each $p \in \mathbf{P}_N$, Y_p is the set of linear subspaces of $\mathbf{P}_N$ of dimension $N - n - 1$ containing p, then if for every $x \in X$ there exist a $p(x) \in \pi(x)$, where $\pi(x)$ is the linear subspace of $\mathbf{P}_N$ corresponding to x, so that

$$T_{\pi(x)}\left(Y_{p(x)}\right) \subset T_x(X) \tag{0.2}$$

then each irreducible component of X comes by taking a variety $W_k \subset \mathbf{P}_N$ of dimension $k \geqslant n$ and taking

(0.3)

$$X = \left\{ \pi \mid \pi \cap W \neq \varnothing \text{ and } \dim\left(\pi \cap T_w(W)\right) \geqslant k - n \text{ for some } w \in \pi \cap W \right\}$$

The case $k = n$ gives the Chow form of W_n. This case can be recognized as follows: the subspaces $T_{\pi(x)}(Y_{p(x)})$ give a codimension n distribution on X. The hypersurface X is a Chow form if and only if this distribution is integrable in the sense of Frobenius.

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