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JOURNÉ'S COVERING LEMMA AND ITS EXTENSION TO HIGHER DIMENSIONS

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Introduction. Suppose T is an integral operator on R, bounded on L^2, which is defined by

$$Tf(x) = \int k(x, y) f(y)\, dy.$$

If the kernel k satisfies

$$\int_{|x-y|>\rho|y-y_1|} |k(x, y) - k(x, y_1)|\, dx \leqslant c\rho^{-\delta} \tag{0.1}$$

for some $\delta > 0$, and if C_T is the smallest constant such that (0.1) holds, then the Calderón-Zygmund norm of T is defined by

$$|T|_{\mathrm{CZ}} = \|T\|_{L^2} + C_T.$$

The classic example of such a T is the Hilbert transform. It is well known that operators of this type map $H^p(\mathsf{R})$ to $L^p(\mathsf{R})$ for $0 < p < \infty$. Similarly, operators which satisfy (0.1) but with respect to differences in the x-variable are known to map L^∞ to BMO(R).

In [4], Journé defines a class of Calderón-Zygmund operators on product domains $\mathsf{R}^n \times \cdots \times \mathsf{R}^m$ and proves that such operators map L^∞ to BMO($\mathsf{R}^n \times \cdots \times \mathsf{R}^m$). Because the defining condition for BMO($\mathsf{R}^n \times \cdots \times \mathsf{R}^m$) is not a direct generalization of the one-variable condition, the proof of this fact requires new ideas and methods. For a product domain with two factors, these ideas (of [4]) were synthesized into a geometric "covering" lemma (also due to Journé) for rectangles in R^2. By combining this lemma with the atomic decomposition for $H^p(\mathsf{R}^n \times \mathsf{R}^m)$, R. Fefferman in [2] has extended Journé's results. There it is shown that Calderón-Zygmund operators satisfying the product form of condition (0.1) map $H^p(\mathsf{R}^n \times \mathsf{R}^m)$ to L^p for $0 < p \leqslant 1$. Our aim in this note is to

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