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BOUNDARY VALUE PROBLEMS FOR WHITTAKER FUNCTIONS ON REAL SPLIT SEMISIMPLE LIE GROUPS

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§0. Introduction. My motivation is to study the spaces of Whittaker functions of real semisimple Lie groups from the view point of the systems of differential equations with regular singularities. Whittaker functions play an important role in the development of the Hecke theory for $GL(n)$. They were first introduced for the principal series of Chevalley groups by H. Jacquet [7]. He studied some homomorphisms (so-called Whittaker Integrals or Jacquet's Whittaker vectors) of the spaces of the principal series to the spaces of the Whittaker functions. G. Schiffmann studied them for linear algebraic groups of real rank one [21]. Later M. Hashizume [5] and F. Shahidi [23] studied generalizations. We may interpret a Whittaker integral as an integral operator with a distribution kernel, which is a homomorphism of a principal series representation to the space of Whittaker functions. Conversely for a quasi-split real semisimple Lie group such a homomorphism coincides with the Whittaker integral up to some scalar factor. See [24], [23], and [21]. N. Wallach [28] characterized the Jacquet's Whittaker vector in terms of moderate growth of the associated K-finite Whittaker functions on G. However in an algebraic context the dimensions of the spaces of Whittaker vectors for the Harish-Chandra modules corresponding to principal series are always the order of the little Weyl groups for quasi-split real semisimple Lie groups. This result was shown by Casselman and Zuckerman for $Sl(n, \mathbf{R})$ and B. Kostant established the general result in [12]. Using differential

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