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MAXIMAL FUNCTIONS AND FOURIER TRANSFORMS

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§1. Introduction. During the last decade, considerable attention has been devoted to the use of the Fourier transform to study problems of pointwise convergence for different kinds of averages. The method, started by E. M. Stein, consists in majorizing the maximal operator by a suitable g-function whose L^2 behavior is, by orthogonality arguments (e.g., Plancherel's theorem), easy to control. A typical example is the maximal spherical mean

$$\mathscr{M}f(x) = \sup_{t>0}\left|\int_{|y|=1} f(x - ty)\, d\sigma(y)\right|.$$

Stein's theorem (see [6]) says that, for $n \geqslant 3$, $\mathscr{M}$ is bounded in $L^p(\mathbf{R}^n)$ iff $p > n/(n-1)$. Quite recently, J. Bourgain [1] has succeeded in extending this result to $\mathbf{R}^2$.

We wish to present here some general results of L^p boundedness for maximal operators $T^*f = \sup_{t>0}|T_tf|$, where T_t is obtained by dilation of a fixed multiplier transformation $T = T_1$. The range of p's depends only on the decay at infinite of the multiplier (and some of its derivatives). Our first theorem includes the results previously obtained for maximal averages over hypersurfaces in [3], [7], [9], with a simpler proof, and can also be applied to more general measures. A second theorem deals with T^* when the multiplier transformation T is not a positive operator. A major drawback of both theorems is that no result is obtained when the decay at infinity is only $|\xi|^{-a}$ with $a \leqslant 1/2$; thus, Bourgain's result mentioned above is not captured into this general scheme.

The fact that two different statements (one for measures, another for arbitrary multipliers) would be necessary became clear to me after a conversation with C. Sogge. I am also indebted to A. Carbery for a discussion which brought to light some of the technical details in the proof of theorem A.

§2. Statement of results. Given a multiplier $m \in L^\infty(\mathbf{R}^n)$, we define the operators $\{T_t\}_{t>0}$ by $(T_tf)^\wedge(\xi) = \hat{f}(\xi)m(t\xi)$, and we wish to study the boundedness properties of $T^*f(x) = \sup_{t>0}|T_tf(x)|$ (which is well defined a priori for Schwartz functions).

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