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ENTIRE HOLOMORPHIC CURVES IN SURFACES

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Introduction. Green and Griffiths have conjectured that every entire holomorphic mapping of C into a surface X of general type is algebraically degenerate, i.e., its image lies in a proper algebraic subvariety of X. For surfaces with irregularity (the dimension of $H^0(X,\Omega^1)$) greater than 2, this is a consequence of Bloch's conjecture, which was proved for surfaces by Ochiai [O] and for higher-dimensional varieties by Green and Griffiths [GG]. We show in this paper that their conjecture also holds for certain surfaces with irregularity 2.

A complex torus which contains no nontrivial proper complex subtorus is called *simple*. Our main result is the following:

THEOREM. *Let X be a smooth algebraic surface of general type, A a simple 2-dimensional abelian variety, and $\pi: X \to A$ a surjective holomorphic map. If $f: \mathbf{C} \to X$ is an entire holomorphic mapping, then $f(\mathbf{C})$ must lie in a fibre of π.*

An entire holomorphic mapping of C into a complex analytic space is also called an *entire holomorphic curve*. Brody [BR] has shown that a compact complex manifold containing no nonconstant entire holomorphic curves must be hyperbolic. If π above is a finite map, then by Brody's theorem, X is hyperbolic.

Let X be a smooth algebraic surface with irregularity 2. The Albanese variety A of X is a 2-dimensional abelian variety, and the image of the Albanese map

$$\pi: X \to A$$

generates A as a group. If $\pi(X)$ is a curve C in A, then C is smooth and has genus 2 [B, Proposition V. 15]. In this case it is easy to see that any entire holomorphic curve

$$f: \mathbf{C} \to X$$

is algebraically degenerate, since the map

$$\pi \circ f: \mathbf{C} \to C$$

must be constant. Combining this fact with the theorem above, we have the following:

COROLLARY. *Let X be a smooth algebraic surface of general type with irregularity $q = 2$, whose Albanese variety is simple. Then every entire holomorphic curve $f: \mathbf{C} \to X$ is algebraically degenerate.*

In the author's thesis [G] some criteria are given for a map $f: \mathbf{C} \to X$ to be

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