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MAXIMAL OPERATORS RELATED TO THE RADON TRANSFORM AND THE CALDERON–ZYGMUND METHOD OF ROTATIONS

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1. Introduction. The following maximal operator has arisen in work of C. Fefferman [17] and A. Córdoba [10, 11] on the L^p boundedness of the Bochner–Riesz spherical summation multipliers in R^n. Let $\delta > 0$ be small and denote by $\mathscr{R}_\delta$ the collection of all rectangular parallelopipeds in R^n, regardless of orientation, which contain the origin and have one side of length r and $n-1$ sides of lengths δr, for all $r > 0$. Define

$$M_\delta f(x) = \sup_{R \in \mathscr{R}_\delta} |R|^{-1} \int_R |f(x-y)|\, dy.$$

The fundamental question concerning M_δ is whether the inequality

$$\|M_\delta f\|_n \leqslant C|\log \delta|^\beta \|f\|_n \tag{1.1}$$

holds for some β, $C < \infty$ as $\delta \to 0$. Córdoba [10] has established this when $n = 2$, and has obtained certain partial results in higher dimensions. Our first result is a sharper but still partial one. First, let us reformulate (1.1) by conjecturing that

$$\|M_\delta f\|_p \leqslant C|\log \delta|^\beta \delta^{-\alpha} \|f\|_p, \qquad 1 < p \leqslant n \tag{1.2}$$

for some β, $C < \infty$ depending only on n and p, where $\alpha = n/p - 1$.

Since M_δ is bounded by $C\delta^{1-n}$ times the Hardy–Littlewood maximal function, (1.2) follows from (1.1) by interpolating between $p = 1$ and $p = n$. Conversely, this power α is best possible, as may be seen by taking f to be the characteristic function of a ball.

Proposition. *The inequality* (1.2) *holds in* R^n *for all* $1 < p \leqslant (n+1)/2$.

For $p \leqslant 2$, this is the partial result of Córdoba [11] alluded to above.

What is striking about this proposition is its relationship to the best bounds currently known concerning Bochner–Riesz multipliers. In dimension $n \geqslant 3$ the full conjectured mapping properties of those multipliers are known to hold for all $p \geqslant 2(n+1)/(n-1)$. Work of Córdoba and Carbery [3, 4] suggests strongly that

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