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THE EXPLICIT RECIPROCITY LAW IN LOCAL CLASS FIELD THEORY

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§1. Introduction. In this paper we prove an explicit reciprocity law that was conjectured by R. Coleman [C2]. It generalizes the explicit reciprocity laws of Artin–Hasse, Iwasawa and Wiles (see bibliography) by giving a *complete* formula for the norm residue symbol on Lubin–Tate groups. We also make some remarks about a larger class of Lubin–Tate groups to which our method applies, and the "Kummer theory" of such groups.

Let k be a finite extension of $\mathbf{Q}_p$. Let π be a uniformizer of k, $\mathcal{O}$ and $\not{p}$ its valuation ring and ideal, and $\kappa = \mathcal{O}/\not{p}$ the residue field. Let q be the number of elements in κ.

Let $\mathscr{F}_\pi$ be the collection of power series $l(X)$ in $\mathcal{O}[[X]]$ satisfying

$$\text{(i)} \qquad l(X) = \pi X + \cdots$$

$$\text{(ii)} \qquad l(X) \equiv X^q \bmod \pi.$$

As is well known, Lubin and Tate associated with any $l \in \mathscr{F}_\pi$ a certain one dimensional formal group F_l over $\mathcal{O}$. Write $[+]_l$ for its addition and $[a]_l$ for the endomorphism whose differential is a. Thus $l = [\pi]_l$. When l varies, the groups F_l are isomorphic to each other over $\mathcal{O}$. If we let π vary too, any two Lubin–Tate groups become (weakly) isomorphic over $\mathcal{O}_K$, where K is the *completion of the maximal unramified extension* of k.

The π^n-division points of F_l form a cyclic $\mathcal{O}$-module of order q^n denoted W_l^n. We let $\tilde{W}_l^n = W_l^n - W_l^{n-1}$ be the *primitive* π^n division points. Then the tower of fields* $k_\pi^n = k(W_l^n)$ is totally ramified abelian over k, $[k_\pi^n : k] = (q-1)q^{n-1}$, and any element of $\tilde{W}_l^n$ is a prime element of k_π^n. As the notation suggests, k_π^n depends on π, but not on $l \in \mathscr{F}_\pi$. Let $W_l = UW_l^n$.

The *Kummer pairing*

$$(\ ,\ )_{n,l} : F_l(\not{p}_n) \times (k_\pi^n)^\times \to W_l^n \tag{1}$$

($\not{p}_n$ denotes the valuation ideal of k_π^n) is defined as follows. For $\alpha \in \not{p}_n$ and $\beta \in (k_\pi^n)^\times$ choose a in the algebraic closure of k such that $[\pi^n](a) = \alpha$. Let σ_β be the Artin symbol of β. Then (dropping the reference to l)

$$(\alpha, \beta)_n = \sigma_\beta(a)[-]a.$$

It is well defined, $\mathcal{O}$-linear in the first variable and linear in the second.

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*Our notation differs from [W] and [C2], where k_π^n is indexed by $n-1$, etc.