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PROGRESSING WAVE SOLUTIONS TO CERTAIN NONLINEAR MIXED PROBLEMS

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1. Introduction. The study of the propagation of singularities for sufficiently smooth solutions of nonlinear hyperbolic equations began with the work of Bony [4], Rauch [13], and Lascar [9]. In general, anomalous singularities not present in the linear theory will arise, as shown in Rauch–Reed [14], and Beals [1]. These additional nonlinear singularities will in all dimensions be weaker than those for the corresponding linear problem. In one space dimension their location is sharply limited (Rauch–Reed [15]) even for very general initial data. But in higher dimensions it follows from the results in [1] that additional smoothness assumptions on the data or on the solution in the past are necessary in order to limit the spreading of singularities. An appropriate condition is that of "conormality" with respect to certain hypersurfaces. It has been proved in Bony [5], [6], [7] and Melrose–Ritter [10], that regularity is preserved for certain interactions of conormal waves; see also Rauch–Reed [16] and Beals [2].

Nonlinear hyperbolic problems on domains with boundary have also been considered. In one space dimension, anomalous singularities again appear in a controlled fashion, as shown in Berning–Reed [3] and Oberguggenberger [12]. And in all dimensions the nonlinear singularities will be weaker than the linear ones (Sablé-Tougeron [17]). In this paper we consider the propagation of regularity when the solution to the mixed problem is conormal with respect to a single characteristic hypersurface in the past. We show that if the hypersurface hits the boundary of the domain transversally, and only one reflected characteristic hypersurface issues from the intersection, then the solution will be conormal with respect to the union of these surfaces.

The proof involves a commutator argument, as first used in Bony [5]. The idea is as follows: Let $\mathscr{M}$ denote the space of smooth vector fields tangent to the hypersurfaces and to the boundary of the domain, and suppose that the commutators of the hyperbolic operator P with the elements of $\mathscr{M}$ can be expressed in terms of P and $\mathscr{M}$. (In the absence of the boundary, this is the "characteristic completeness" property of [10].) Then classical energy estimates and a bootstrap procedure applied to the derivatives of a solution with respect to elements of $\mathscr{M}$ allow conclusions about the smoothness in the future given smoothness in the past. We show using ideas similar to those in [5] that $\mathscr{M}$ satisfies the appropriate property. The induction argument then involves working first microlocally in the space generated by the conormals to the characteristic

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