Vol. 53, No. 1 DUKE MATHEMATICAL JOURNAL © March 1986

ON A PROBLEM OF CHISINI

F. CATANESE

§0. Introduction. The objects of this note are generic multiple planes, defined according to the following

Definition 1. A multiple plane is a pair (S, f) where S is a compact smooth connected complex surface and f is a finite holomorphic map $f: S \to \mathbf{P}^2 = \mathbf{P}^2_{\mathbf{C}}$. (S, f) is said to be generic if the following properties are satisfied:

(1i) the ramification divisor R of f is smooth and reduced

(1ii) $f(R) = B$ has only nodes and ordinary cusps as singularities

(1iii)* $f_{|R}: R \to B$ has degree 1.

Moreover, two multiple planes (S, f), (S', f'), are said to be isomorphic if there is an isomorphism $\phi: S \to S'$, and a projectivity $g: \mathbf{P}^2 \to \mathbf{P}^2$ such that $f' \circ \phi = g \circ f$, and strictly isomorphic if furthermore $g =$ identity.

Obviously, a necessary condition in order that two multiple planes be isomorphic is then that the two branch curves B, B' be projectively equivalent. Without loss of generality, therefore, we shall consider pairs of generic multiple planes (S, f), (S', f') such that $B = B'$, and we will investigate the problem of deciding whether they are strictly isomorphic. i.e., there does exist an isomorphism $\phi: S \to S'$ such that $f' \circ \phi = f$. Such a problem was considered by Chisini (cf. [C]) who conjectured that two generic multiple planes with the same branch curve would be strictly isomorphic "under some suitable conditions of generality."

The problem has a negative answer in general (contrary to the statement of the main theorem of [L]), as it is shown by a very nice example of Chisini himself in [C] (a previous example given by B. Segre in [S] yields a nongeneric triple plane).

Our result consists in giving a necessary and sufficient condition for strict isomorphism: at the end of the paper we shall discuss Chisini's example to illustrate our theorem. To explain our condition, we need a technical definition.

Definition 2. A marked curve $(C, p_1, \ldots, p_\gamma)$ consists of a (compact connected complex) curve C, together with an ordered set of γ points of C.

A marked line bundle $\mathscr{L} = (L, h_1, \ldots, h_\gamma)$ on $(C, p_1, \ldots, p_\gamma)$ consists of the datum of a holomorphic line bundle L on C, and of isomorphisms h_i $(i = 1, \ldots, \gamma)$ of the fibre L_{p_i} of L over p_i with $\mathbf{C}$.

Partly supported by M.P.I. and C.N.R. (contract no. 82.00170.01).

*As pointed out by the referee, if $f: S \to \mathbf{P}^2$ is generic, and $h: S' \to S$ is finite and etâle, $f \circ h: S' \to \mathbf{P}^2$ satisfies (1i), (1ii), but not (1iii).

Received February 26, 1985. Revision received September 16, 1985.