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ON THE DEFORMATION THEORY OF CLASSICAL SCHOTTKY GROUPS

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Imagine a finite collection of circles $\mathscr{C} = \{C_j\}$ in the Riemann sphere S^2, whose interiors are all disjoint. Suppose we are given a division of $\mathscr{C}$ into pairs $\mathscr{C} = \bigcup_i \{C_{i_1}, C_{i_2}\}$ and for each pair a Möbius transformation $\gamma_i : S^2 \to S^2$ taking the interior of C_{i_1} to the exterior of C_{i2}. Then the group Γ generated by the γ_i is a discrete group of Möbius transformations, abstractly isomorphic to a free group, and is called a classical Schottky group. A fundamental domain for the action of Γ is given by the common exterior to all the circles.

In [2], we began a study of the geometry of classical Schottky groups, based on the previous work of Phillips–Sarnak [3]. In [3], the authors observed that, given such a group, certain geometric quantities could be estimated by replacing Γ by the group Γ' generated by reflections in the circles $\{C_i\}$. In this way, one may obtain estimates based on the circles themselves, and not on how they are paired, which should simplify the problem.

To that end, let us capture the combinatorial data of how these circles fit together, in the following way: given $\mathscr{C}$, we construct a polyhedral division of the sphere by placing one vertex at the center of each circle, and one edge joining two centers whenever the corresponding circles are tangent.

For the applications we have in mind, it is always reasonable to add more circles to the collection $\mathscr{C}$. Furthermore, $\mathscr{C}$ is particularly nice if the region exterior to all the circles is a disjoint union of curvilinear triangles, or, equivalently, if the polyhedral decomposition of the sphere mentioned above is a triangulation.

Thurston has observed that such particularly nice configurations are conformally rigid, by the Mostow Rigidity Theorem [5]. That is to say, given a triangulation of S^2, there is at most one configuration of circles, up to translation by a Möbius transformation, which gives rise to this combinatorial data. He also proved the deeper result [5] that any such triangulation is realized by some configuration of circles, which therefore is unique.

It would therefore seem that such particularly nice configurations are rather sparse (there are only countably many of them). One result of this paper is that actually the opposite is true—in some sense, such particularly nice configurations are dense.

To understand this, let us add circles to our configuration so that all the components of the exterior of all the circles are as simple as possible. Elementary

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