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LOCAL COEFFICIENTS AS ARTIN FACTORS FOR REAL GROUPS

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Introduction. The purpose of this paper is to prove the equality of certain local coefficients of arithmetic significance which were attached to representations of quasi-split real reductive algebraic groups in [27] with their corresponding Artin factors attached by local class field theory [21]. As a consequence, we establish an identity satisfied by certain normalized intertwining operators. It seems to be useful in applications of the trace formula [1, 29].

More precisely, let G be the group of real points of a quasi-split reductive algebraic group over $\mathbf{R}$. Let Δ be the set of simple roots defined by a fixed minimal parabolic subgroup $P_0 = M_0A_0U$ of G. Fix $\theta \subset \Delta$, and let $P = P_\theta$ be the corresponding standard parabolic subgroup of G and write $P = MAN$ for its Langlands decomposition. Fix a nondegenerate character χ of U and let $(\sigma, H(\sigma))$ be an irreducible admissible χ-generic Banach (in particular χ-generic unitary) representation of M (cf. Section 1). Given $\nu \in \mathfrak{a}_{\mathbf{C}}^*$, the complex dual of the Lie algebra of A, let $I(\nu, \sigma, \theta)$ be the continuously (quasi-unitarily, if σ is unitary) induced representation $\mathrm{Ind}_{P\uparrow G}\sigma \otimes e^{\nu}$, and let $V(\nu, \sigma, \theta)$ be its space (Section 0). Then $V(\nu, \sigma, \theta)_\infty = V(\nu, \sigma_\infty, \theta)$.

Now, let W be the Weyl group of A_0 in G. Choose $\tilde{w} \in W$ such that $\tilde{w}(\theta) \subset \Delta$. Let N^- be the unipotent group opposite to N. Define $N_{\tilde{w}} = U \cap wN^-w^{-1}$, where w is a representative of $\tilde{w}$ in G. For $f \in V(\nu, \sigma, \theta)_\infty$, define

$$A(\nu, \sigma, w)f(g) = \int_{N_{\tilde{w}}} f(gnw)\,dn \qquad (g \in G)$$

(cf. (3.1) and (3.2) of Section 3). The convergence and meromorphic continuation of $A(\nu, \sigma, w)f$ has been studied by Knapp and Stein in [12, 13] (for minimal P see also Schiffmann [24]).

The representation $I(\nu, \sigma, \theta)$ is χ-generic. Let $\kappa(\nu, \sigma)$ be the canonical Whittaker functional attached to $V(\nu, \sigma, \theta)_\infty$ by relation (2.2) of Section 2. Then the local coefficient $C_\chi(\nu, \sigma, \theta, w)$ is a complex number defined by

$$\kappa(\nu, \sigma)(f) = C_\chi(\nu, \sigma, \theta, w)\kappa(\tilde{w}(\nu), \tilde{w}(\sigma))(A(\nu, \sigma, w)(f)),$$

$\forall f \in V(\nu, \sigma, \theta)_\infty$ (cf. Section 3 and [27]).

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